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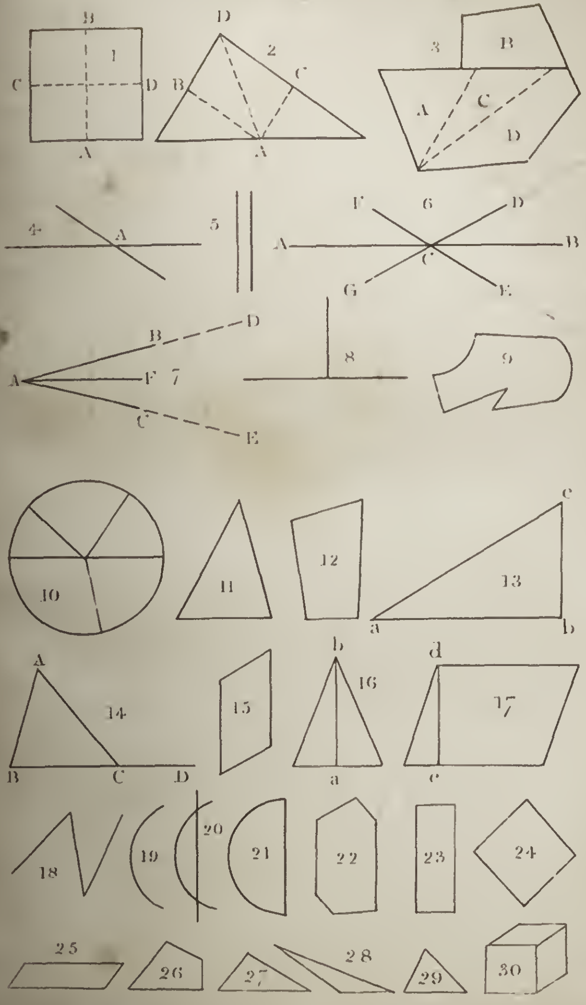

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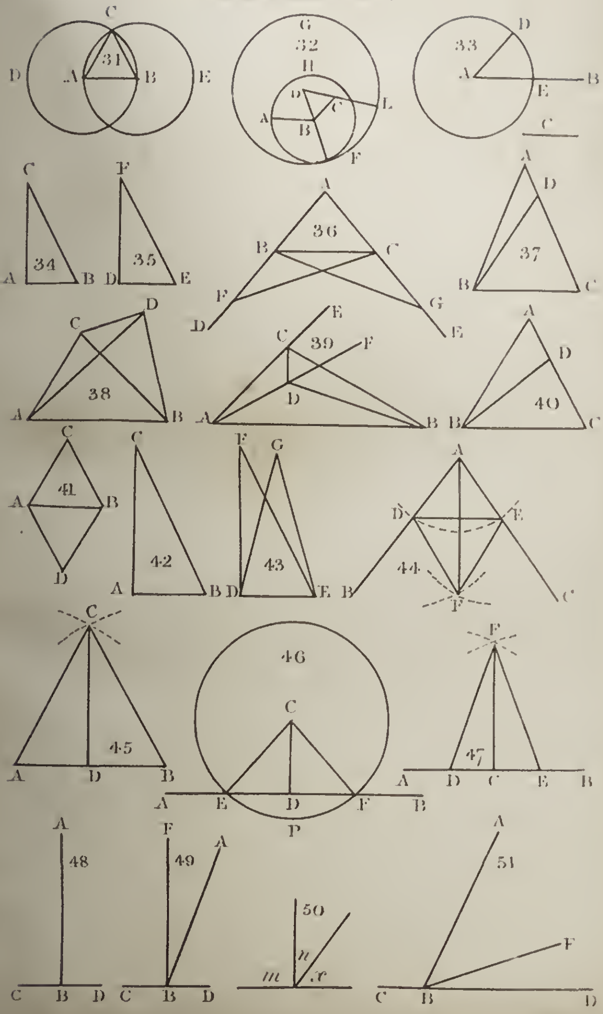

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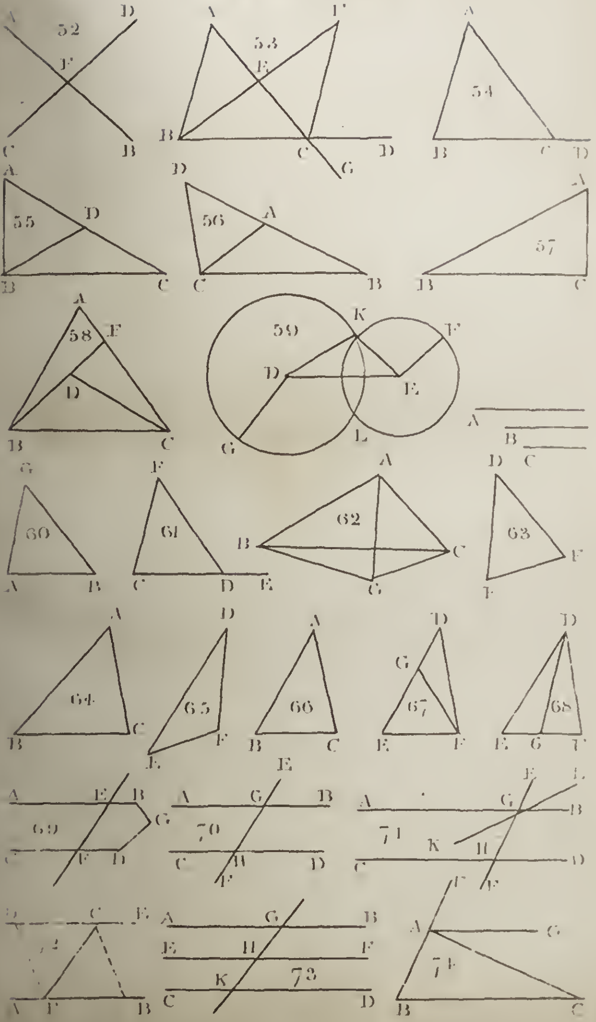

Preface of 1866.

Experience has shown that this book was some utility though open no doubt to many objections. For the sake of such service I shall not deprecate any adverse criticism

~~not be found to deprecate~~ but shall keep in recollection the prudent advice of Helicane, and

> With patience bear
> Such griefs as I do lay upon myself.

The book is open to the fault of being too crude & too confident judged from the ~~[illegible]~~ point of scholasticism. My friend Mr Knight Hunt, who preceded Mr Weir as Editor of the Daily News, suggested to me the title "Mathematics no Mystery", as ~~explain~~ indicating in a rough way the ~~[illegible]~~ enquiries the book was intended to satisfy.

The chief object in view has been to furnish some practical explanation

velopment of the argument in favour of this science as a means of mental discipline, guarded from the arrogance and exaggeration which have so often destroyed its efficiency. 6. An exposition of Mathematical logic, and the true principles of Science, illustrating the value of the one and the simplicity ~~of the other. 7. The enforcement~~ of Natural Geometry—~~resting it on the foundation of the common understanding~~, distinguishing its Beauties and Uses, adapting it to the wants of the many, to the capacities of the young, to the aptitudes of the uninitiated, and the exigencies of men of business—the people of little time and a definite purpose.

~~G. J. H.~~

The word EUCLID, on the title page, contains the Diagrams of six of the most interesting and celebrated Propositions of Euclid. The number of each Proposition, and the number of the Book in which it is found, are placed at the foot of each letter. The Diagrams in E, U, C, occurring in the First Book, the reader will discover, in the following pages, to what they relate. The Diagram in L is employed to prove that all the angles that can be made in a semicircle are right angles. The Diagram in I belongs to the Proposition respecting the proportion between triangles and parallelograms of the same altitude. The Diagram in D is the geometrical foundation of the Rule of Three. ~~It seemed the most appropriate tribute to Euclid to associate his works with his name.~~

G. J. H.

What it is that it is worthwhile doing well

CONTENTS.

What it is that it is worth while doing wel

The First Book

INTRODUCTION TO THE ~~MATHEMATICS.~~

CHAPTER I.

~~DISCOURAGING INFLUENCE OF A CERTAIN POPULAR MAXIM OVER THE PURSUIT OF LEARNING IN GENERAL, AND MATHEMATICS IN PARTICULAR.~~

'WHATEVER is worth ~~while~~ doing at all, ~~it~~ is worth ~~while~~ doing well,' is a maxim of deserved reputation: and *when* applied in its proper sense, of great service. ~~But, adopted as this has been, without due regard to its exact import, it may be questioned whether any maxim ever proved so serious a discouragement to the majority of learners.~~ What ~~it~~ is worth ~~while to~~ do at all, it is, unquestionably, sound wisdom and true economy to do well. But *what* ~~it~~ is worth ~~while to perform~~, and the *measure* of the undertaking, are to be decided by the wants and circumstances of the person engaging in the pursuit. Instead of this being done, we have those who, on feeling interested in the investigation of any branch of learning, conclude it to be incumbent on them to entirely explore it, whatever be its nature or extent, and, wanting time to accomplish this, often an extravagant task, remain in entire ignorance of that of which it was probably important they should know something.

ing/ doing/

The person destined for the profession of Chemistry, of necessity must acquaint himself with the works of ~~Priestley, Davy, Dalton, Liebig, Faraday, Daniels, and other~~ eminent men in that department of knowledge: but the general inquirer will sufficiently in-

the

same letters as in places not-cancelled

form himself on this subject from Chambers' 'Rudiments' of the science. In like manner, a person intended for the Medical profession has ~~a wide class of~~ *many* authors ~~before him~~, with whom he must become conversant, while the artizan would be ~~considered~~ well informed who was familiar with the works of Southwood Smith, ~~and~~ Andrew Combe. Now, *what* it is proper for the Chemist and Medical student to undertake, is no less than the whole range of knowledge belonging to each of their respective professions—and *this* it is worth their while to master well. But the general inquirer ~~and the~~ *or* artizan, being differently circumstanced, require ~~only~~ such a general insight ~~into~~ *of* these subjects as shall enable ~~them~~ *him* to converse thereon with propriety, to understand books in which they are introduced, and to apply the plain and popular rules to the business, enjoyment, and preservation of life.

The ~~judicious~~ rule of acquisition is this. After mastering the knowledge proper to individual profession, we next have to seek general knowledge: for he who should ~~essay~~ *attempt* to sound the depths of every subject, would find life exhausted before he had completed a tithe of his task. ~~A course which would condemn to the unremitting toil of hoarding information, and exclude for ever from the pleasure of its application.~~

~~Unmindful of considerations of this kind,~~ It is commonly thought, that whenever Geometry is proposed for investigation, (since whatever is worth while doing at all is worth ~~while~~ doing well), nothing less than Dr. Simpson's Eight Books of Euclid, his namesake's Trigonometry, Bonnycastle's Mensuration, Bridge's Algebra, *the works of De Morgan & and Potts,* and a host of ~~scientific~~ *relevant* treatises, must necessarily be ~~entered upon~~ *mastered*. And as it is only the time of professed students that will allow of this, the

entire subject is generally neglected.* In what belongs to a man's profession, his duty, or his chosen study, there must be no superficialness. ~~The professional astronomer must be master of Newton—the~~ mariner, of Thomas Simpson—the architect and civil engineer, of Euler and the Bernouillis, and a certain class of able geometricians. So much of geometry as may belong to any branch of mechanics, must be mastered by all who would be proficients in such branch —whatever be ~~the formidable array of works to be encountered.~~ But ~~under other circumstances~~, what is ~~chiefly~~ wanted is such a *general* acquaintance with the subject as shall enable a person to distinctly understand the nature and application of Mathematics—the process of geometrical reasoning—the meaning of the technical terms now so frequent in the scientific lecture room, and in treatises on Mechanics.†

by the unprofessional student

* Of forty artizans who commenced the study of Mathematics, under the impression, in the Birmingham Mechanics' Institute (1837-9) scarcely one-eighth pursued the subject to a satisfactory issue: and the few who did, pursuing it at the usual length, had to make large abstractions from the hours of recreation and rest, and, to a great extent, paid their health as the penalty of their application.

† Greek and Latin, Aristotle's logic and classical versification, quadratic equations, conic sections, the differential calculus, are very good things.—They are essential to the fame of a Parr or a Porson, a Herschel or a Whewell. These acquisitions are, doubtless, amongst the greatest triumphs of the human understanding, and are calculated to raise a few—perhaps one in a hundred—to distinction; but upon the minds of the remaining ninety-nine they produce no sort of impression. They do not do much mischief to such persons in themselves, but they are of incalculable detriment, and the industry which they absorb to no available purpose. Ten years of youth—the most valuable and important period of life—are wasted in studies which, to nineteen-twentieths of the persons engaged in them, are of no use whatever in future years.—*Blackwood's Magazine, Nov.* 1845.

let

that they must know every thing

Much has been said of the qualities of calmness, of clearness, coherency, and cogency—the reputed traits of a geometrical mind: and a general desire of acquaintance with the writings of Euclid is felt on this account. ~~Now,~~ These points ~~enumerated~~ may be learned in a very satisfactory manner from the First Book of Euclid, which six weeks' application will master. ~~On this account the First Book only is introduced in this Work—that Book constituting a complete introduction to geometrical science.~~

The usual works on 'Geometry,' and those under the name of 'Euclid's Elements,' have been mostly intended for professional persons, or teachers, as few besides were likely to buy them. Sometimes they have been issued by individuals who merely sought geometrical reputation. In both cases such works would naturally contain much which the general student would never want. ~~Now that enquirers have multiplied, the pedantry of~~ the schools has been brushed away in the bustle of practical life, and zeal of learning broken through the barriers of caste — the popu~~lace claim to have opened to them the portals of Geometry~~. Euclid, by the unanimous consent of learned men, is the keeper of the gate. His First Book is the entrance to the Temple~~—and it is strange that none among the moderns have selected this vestibule for the admission of the public to view the museum of the exact sciences~~.

of the

Proclus, the last Greek geometer, seems to have had this idea. He alone, of the ancients, is recorded to have paid distinguished attention to the First Book of Euclid. He left a special commentary upon it, abounding in curious observations on the metaphysics of Euclid, and ~~with~~ curious facts connected with the full science.

The desirability of recasting our educational trea-

of Geometry

tises the commercial habits of the age leave beyond doubt. The industrious classes are they who ose/ most need the acquisition of any logical power which Geometry can impart, and their means of leisure to pursue such a study are few and far between. Actuated, probably, by convictions of this kind, Sir John Herschel has composed a work on Astronomy, intended to answer for the astronomical, such purposes as are here contemplated for ~~the~~ geometri~~cal public~~. ~~Indeed, many distinguished men have turned their attention to the same object in their respective sciences. Of the fitness of the single book of Euclid for the accomplishment of the ends here specially in view, the~~ [redacted] ~~will be able to judge from the execution of these pages~~. Since the ~~writer~~ first arranged re/page, ~~them,~~ a coincident opinion has been very distinctly were expressed upon the subject.—

'To young readers, about to enter on the study of Astronomy, and who seek only to get such a mere general knowledge of it as may satisfy their own minds, we would say in the first place, acquire a knowledge, however slight it may be, of the elements of Mathematics. Your mind may not be of a mathematical turn, and there may not be the slightest prospect of your deriving any positive advantage from posing your brains with the "*First Book of Euclid.*" No matter; try and go over it; it is worth your while. You cannot stir in Astronomy without knowing something of the properties of the circle and triangle. He, therefore, who wishes to comprehend the "reasons" on which Astronomy is based, will acquire this preliminary knowledge, without which it is useless for him to enter upon the study. After he has acquired it, and after he has studied an Astronomical work, he may be far—very far, indeed, from having the smallest pretension to the name of Astronomer,

but he will be in possession of a few of the "fundamentals" of the science; and will stand on the same platform with the Astronomer himself.'*

Shakespeare

The world has been favoured with the Beauties of ~~Beattie~~, of Byron, Scott, ~~and of its great benefactors in literature and song, and the public who could never acquaint themselves with the entire productions of these authors, have become familiar, through such convenient epitomes, with their merits and their graces.~~ There is ~~That there are both romance and~~ beauty of a stern kind in the wonderful writings of Euclid, ~~no man of information or taste will deny.~~ His works were the torch which lighted the sciences to their earliest discoveries. They revealed ~~to us~~ the world of form and the hidden secrets of figure, and enlisted lines and angles in the eternal service of man. ~~The deep enthusiasm which some have felt for Geometry will be generally shared in the current diffusion of intelligence.~~ A~~n important~~ connection is beginning to be established between the principles of science and morality. More than one philosopher is found to acknowledge, that 'The axioms of Physics translate those of Ethics.'† The desire of aiding to some extent the development of this fact was not one of the least reasons which has dictated the compilation of the 'Beauties and Uses of Euclid.'

* London Saturday Journal.

† Emerson.

CHAPTER II.

~~IMPORTANT~~ DISTINCTIONS PERTAINING TO MATHEMATICS.

Mathematics is the science of quantity and magnitude. It is usually defined as that which treats of whatever can be numbered or measured.* Mathematics embraces all modes of procedure, and includes all the arts and sciences employed in determining magnitude and quantity. Quantity is understood to include the distinctions of weight and measure. Magnitude is the consideration of size.

Geometry, though a very important science, is but a branch of the Mathematics. Geometry treats of the properties of lines, angles, surfaces and solids. The term Geometry is applied to Mensuration, to Trigonometry, Conic Sections, and to every department of investigation in which lines and angles, surfaces and solids, are chiefly concerned.

Arithmetic, the art of figures—and Algebra, the science of symbols, constituting the lower and higher departments of calculation, are both branches of Mathematics. But Geometry is the principal branch.

~~From these points of definition the relative dis-~~tinctions subsisting among the branches of Mathematics, will easily be perceived, and the careful observer will be enabled to avoid the error of confounding these distinctions. The common mystery attached to the Mathematics has, in a great measure, its rise in ~~this kind of inattention.~~

* Ritchie's Principles of Geometry, page 2.

A person skilled in the use of figures is an Arithmetician. ~~This is his proper appellation~~. If acquainted with that higher branch of calculation in which quantities are represented by letters, his ~~correct~~ designation is an Algebraist. The proficient in Euclid's Elements is a Geometrician. The more precise ancients always styled Euclid ~~the illustrious author of the Elements,~~ the Great Geometer. It is the person who has acquired skill in *all* ~~these~~ branches, ~~and such others as~~ pertain to the determination of magnitude and quantity, ~~who~~ is a ~~properly the~~ Mathematician. As Mathematics includes all these arts, properly, the Mathematician is master of them all.

~~We often, indeed, hear the skilful Algebraist styled a Mathematician. The Trigonometrist and Geometrician often receive the same designation. But correct taste is never guilty of this confusion of terms. Such want of precision is a poor compliment to the precisest of sciences.~~

~~The term Mathematics will never be intentionally employed in this work, except in its comprehensive sense, as explained in its definition. That term on the title page is employed to indicate that an attempt will be made to lay bare its clear import, and by so doing divest it of its reputed mystery; and illustrating (as will be done in various places) the exact nature and influence of its branches, and afford a comprehensive and easy insight ~~into~~ of its nature and application. But chiefly, this work, as its second title expresses, is confined to Euclid—to Geometry.~~

The Elements of Euclid are often designated the Elements of Mathematics. ~~True,~~ The Elements once constituted Mathematics themselves; but since the perfection of numerals and the invention of Algebra by Diophantus, they have sunk into a relative rank. It is true, Geometry gives laws to Arithmetic. The

Second Book of Euclid illustrates the powers of numbers. None of the ancients unfolded the ~~pro-found~~ theory of proportion ~~with such ability~~ as Euclid did in his Fifth Book. The 16th Proposition of the Sixth Book is the foundation of the Rule of Three. ~~The logic of Euclid is, to a great extent, the logic of the Mathematics. But though these cardinal and common qualities enable us to make the First Book an introduction to Mathematics,* we are not, on such ground, warranted in confounding, even for a moment, the lesser with the greater. The Elements of the Mathematics signify the elements of all the branches of the Mathematics.~~ The Elements of Euclid are the ~~elements of~~ always Geometry.

~~Forgetfulness or rather ignorance of these natural distinctions, causes every pupil who comes to the study~~ students of Euclid, ~~to~~ put the ~~idle~~ question 'Is a knowledge of arithmetic requisite before commencing the study of the Elements?' ~~It was my first question, and it has been the first question of my pupils to me~~. No ~~enquiry could be more uncalled for.~~ If the pupil was entering on the study of the entire Mathematics, it is plain he wants no arithmetic to begin with, ~~as the province of Mathematics includes that art. And~~ since it is Geometry ~~it is~~ that teaches the The principles of *science*† of numbers. ~~Its~~

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* The Books of Euclid are a series of graduated steps, and when we have learned the principle of mounting the ladder we can climb to any height. The *First* Book developes these principles. It is in this sense that the First Book is here put for the whole fifteen.

† In another place (Practical Grammar, p, 9.) I have found it necessary to explain what it may be useful to repeat here, that 'an Art is a system of rules, for the performance of any operation—and Science is that which explains the *reasons* on which the rules of art are founded.' The Elements of Euclid being eminently a demonstrative science perform this office

~~agents are lines and angles. It is a distinct~~, entire, and independent science.

But it will be asked by the intelligent reader, if Mathematics comprises not only Arithmetic, Algebra, Differential and Integral Calculus, and Geometry—but also such principles as are employed in the measurement of the earth and the skies, in Physical Geography, Astronomy, Navigation, Architecture, and a variety of other branches, how is the maxim with which Sec. I. opens to be applied by the general enquirer to these departments? The answer is easy.

If the enquirer aims at the eminent distinction of being a Mathematician, he must prosecute each of these branches of study, so as to acquire familiarity with their modes of procedure, and some skill in their application. Nor will this be so difficult as at first sight appears. The nature of each is kindred, and Geometry is the key which will unlock them all—only it requires the application of many years to compass so wide a range of physical knowledge.

The course of the general enquirer is different. Arithmetic being required, to some extent, by every-

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for Arithmetic and several other arts. The Arithmetician and *Practical* Geometer, (a distinction hereafter to be explained) come to Euclid for the reasons of their rules. I am aware that Schiller (according to the *Dublin University Magazine*) makes the great distinction between art and pure science to consist in the element of humanity being always essentially involved in every work of art, and this element being excluded in all that is properly called science. But this distinction relates to high art—to the poetry of art. I have here to do with art in the sense of manipulation—and with science in the sense of systematized and demonstrative knowledge—and in every employment of these terms, allusion is made to the art and science of practice. No disparagement to Schiller's distinction, but ours is the other department.

body, *he* acquires its first principles, and as much of its practice as he is likely to need. If he is curious about Algebra, he may soon gather from Coles's admirable 'Conversations' on that subject, the requisite information—the First Book of Euclid will accomplish the rest. It will introduce him to both Theoretical and Practical Geometry, and to the technical characteristics of Mathematics. It will enable him to understand the *modus operandi* of Newton's Principia. It will familiarise him with the illustrations of Dynamics and Mechanics. After performing the 'Constructions of Euclid, and putting them to the 'Uses' at the end of this book, any person could understand and practice Keith's principles of Mapping, and illustrate his geography by constructing his own maps. We have seen in the quotation in the last chapter, from the *Saturday Journal*, how the First Book of Euclid, if it does not make the young pupil master of Astronomy, it will place him on the same common ground with the Astronomer himself, and familiarise him with the scientific secrets of that fine pursuit. Indeed, if the First Book of Euclid does not make the student a Mathematician, it so qualifies him that he can be a Mathematician whenever he pleases, as it imparts a clear insight of that science which will enable him to extend his studies whenever his duty or taste shall incline him. Thus the general enquirer applies our maxim. He determines what is properly worth *his* attention, and earnestly devotes himself to that, and to nothing else. His business is to *explore*, not to *conquer* the region of Mathematics It is that which ~~it is worth his while to do, and he does *that* well.~~

~~Many~~ people of undisciplined powers, and ignorant of the art of acquiring knowledge, can never move stet
anywhere unless they move profoundly. Such people soon lose themselves. Instead of being the masters of

knowledge, knowledge is the master of them. ~~By being profound out of place, they bring profundity into contempt. It is not within the compass of human power to be profound upon more than a few subjects~~ The poet was somewhat too strict in his limitation, but he approached the truth in declaring—

> One subject only will one genius fit,
> So vast is art—so narrow human wit.

Do not be deterred from the useful course of general investigation, in these two Sections recommended, by the vague charge of superficiality, so often preferred without propriety and listened to without advantage. Many persons live and die in disreputable ignorance because they are determined not to be superficial. Superficiality is a serious imputation when it respects what you ought to know well. But with respect to general knowledge, it is no more criminal to be superficial than to have only five senses, or not to be in half a dozen places at once. It is the condition of humanity only to be able to take a general survey of the wide field of knowledge. The republic of literature has no inapt resemblance to the state of good citizenship, in which a man is expected to know his own business, and to have such an acquaintance only with that of his neighbours as shall give him a laudable interest in their pursuits, and assure him that they are conducive to the dignity of the commonwealth.

~~Of the majority of the branches of learning, we can only hope to master the first principles. And it happens that no work that human wit has ever devised teaches the art of doing this in so striking a manner as does the First Book of Euclid. But more of this in the proper place. We must now introduce the reader to the History of the Mathematics, for which he is prepared. The record of its gra-~~

dual rise out of the wants of men by patient study, and its progress, will further divest him of the prejudice, if any remain, of the popular mystery connected with this science.

CHAPTER III.

HISTORY OF ~~THE RISE AND PROGRESS OF THE~~ MATHEMATICS.*

'The Mathematical Sciences were the first of all among men, if we may believe *Josephus*. He (Bk. I. Chap. 3,) writeth, that the posterity of Seth observed the order of the Heavens, and the courses of the Stars. And lest these inventions should slip out of the knowledge of men, *Adam* having predicted a two-fold destruction of the Earth, one by a Deluge, and the other by Fire, they raised two Columns, one of Bricks, the other of Stone, and inscribed their inventions upon them; and if the Brick one should happen to be destroyed by the Deluge, that of Stone, which would remain, might afford men an opportunity of being instructed, and present to their view the things which had been inscribed upon it. They also say, that that

* Compiled chiefly from Andrew Tacquet's (of the 'Society of Jesus') Elements of Euclid, translated by Whiston—the Introduction to the Elements of Geometry, translated into English from Peyzard, by Phillips—and from the History of the Science prefixed to the Elements of Plane Geometry, by Bell. ~~But, of course, omitting the usually recited lists of the inventions of the 'Fathers of the Science.' If enumerated,~~ who, on a general introduction to Mathematics, would feel enraptured or be enlightened by details of 'The Lair,' 'The Loci,' Pori, Quadratics, and Curves? They are best introduced where they are intelligible.

Stone Pillar, which even in our day is seen in Syria, was dedicated by them. This *Josephus* says, whom I leave to vouch for the story.'*

Geometry must be coeval in its origin with the invention of the square and the compasses, and these being necessary to the rudest operations, were doubtless known in the earliest ages of the world. Geometry first assumed the character of a science, in Greece, about 2,500 years ago, and flourished during ten centuries: then was almost entirely neglected till about 200 years since, when it broke forth in Europe with more than its pristine brilliancy.

The term Geometry is derived from the Greek words *ge* (earth) and *metron* (measurement), literally signifying the *Measurement of the Earth*. But the science thus designated is now so extended, as to be equally applicable to the measurement of the heavens. Aristotle, the famous preceptor of Alexander the Great, ascribes its origin to the Egyptian Priests, and Herodotus, an ancient historian, to the time when Sesostris, who reigned in Egypt, (1500 B. C.)† intersected it by numerous canals, and apportioned the country among the inhabitants.

To Egypt appears chiefly due the honour of having been, if not the birth place, at least the cradle of Geometry, where it received its first culture. That old and mighty kingdom seems to have been the nurse, perhaps the mother, of almost all the useful arts and sciences. Hindoostan and China contend with Egypt for the pre-eminence with respect to Geometry; but by whichever it may have been first cultivated, it

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* Tacquet's Historical Account of the Rise and Progress of Mathematics.

† B. C. Before the reputed Birth of Christ—before the commencement of what is denominated the Christian Era.

was certainly brought into general notice by the Egyptians.

Thales, the reputed father of Greek philosophy, born at Miletum (640 B. C.), is celebrated for being the first who introduced Geometry to Greece, and for the many highly interesting discoveries he made in it: for being able, by geometrical principles, to compute the distance of vessels from the shore, and for having measured* the altitude of the Pyramids of Egypt by means of their shadow, much to the astonishment of Amasis the king, who wondered at his scientific skill. From him Astronomy is said to have made considerable advances—and he founded an establishment in Greece known by the name of the Ionian School. It was in the age of this Philosopher, according to the best informants, that Geometry first began to assume the character of a science. He observed the Equinoxes and Solstices, and was the first who foretold the Eclipse of the Sun. And he foretold to King Cyrus an Eclipse of the Moon. The Electrical properties of amber were first remarked by him, upon which has been built the important science of Electricity.

The first elementary treatise on Geometry is said to have been composed by Anaximander, a disciple of Thales, and Thales's successor in his school.

Next in order comes Pythagoras,† the philosopher

* Thales, one of the seven wise men of Greece, about 600 years before Christ, invented the following method for measuring the height of an Egyptian pyramid. He watched the progress of the sun till his body and shadow were of the same length, and at that instant measured the shadow of the pyramid, which consequently gave its height.—*Lord Kame's History of Man, vol.* 1, *p* 91.

† The supposed inventor of the term 'Philosophy'—signifying *a lover of knowledge.* A term said to have been first applied to Solon, an ancient law-giver.

of Samos, (an island in the Levant, between Greece and Asia), no less distinguished than Thales for the variety and extent of his discoveries. He is said to have travelled into Egypt and India in pursuit of knowledge, to have settled at Tarentum in Italy, (about 550 years B.C.), and to have maintained, in Astronomy, the true system of the world which places the sun in the centre with the planets revolving round it. From this philosopher it was called the Pythagorean System, and was revived two thousand years after his day by Copernicus. Pythagoras was distinguished in all his pursuits by a genius remarkably inventive and assiduous. He possesses no ordinary claim to the honour of posterity. As a mathematician, he was decidedly the first of his time,—as a philosopher, he delivered many curious things concerning God and the human soul, and a great variety of precepts relating to the conduct of life, both civil and political.

It is not certain at this time whether Geometry had been founded into a regular system. If not, considerable advances had been made, and ere a century had elapsed from the age of Pythagoras, Zendorus, a man of great parts, arose, whose writings are the first among the ancients which have survived the wreck of time, one geometrical tract of his having been preserved.

Then came Hippocrates, whose brilliant genius, great diligence, and application, rendered essential service to this science. He was the third who wrote on the Elements of Geometry, but on the appearance of the Elements of Euclid, his treatise was consigned to oblivion.

The origin of the celebrated Platonic School, (400 years B.C.), is considered one of the most important epochs in the history of the science. Plato, its

founder, who is called by way of pre-eminence the 'Athenian Sage,' and who was alike remarkable for eloquence, the poetical sublimity of his imagination, and the accuracy of his mathematical performances, gave to Geometry the form and substance of a complete science, and enriched it with many discoveries. Indeed, so profound a veneration did he entertain for it, that he made it a principal object of instruction among his scholars, and had written over the entrance of his academy, the well known words,—'*Let no one enter here who is ignorant of Geometry.*' Plato was the discoverer of the Conic Sections, (or those curves which are found on the surface of a Cone when cut through in different directions). He found out many of their most remarkable properties, which being made the continual study of his scholars and successors, at length became a distinct science from the common elements, and received the appellation of the *higher or sublime* Geometry.

Such was the versatility of his genius, that he directed his acute and original powers with eminent success to the cultivation of Moral Philosophy, and produced his famous work, entitled the 'Republic,' with the view of imparting to civilised intercourse the character of a science, and giving a philosophical direction to the ever jarring aims and interests of mankind.

The celebrated Aristotle, the successor of Plato, and preceptor to Alexander the Great, and who is justly regarded as one of the greatest men of olden time, made many improvements in the mathematical sciences. After attending the lectures of Plato, he opened a school himself in the Lyceum of Macedonia, which was assigned him by the magistrates, and was the founder (about 350 years B.C.), of the well known Peripatetic school.

Theophrastus, a disciple of Aristotle, composed a complete history of the origin and progress of Mathematics, Astronomy, and Arithmetic, from the earliest periods to his own time. The treatise has been unfortunately lost.

Archytas, of the academy of Plato and of Tarentum, the place in which Pythagoras settled, is famous as the constructor of a flying pigeon, and as the reputed inventor of the pulley and screw. 'He was one of the first,' says Tacquet, 'who brought down the Mathematics to human uses.' Aristœus, who flourished (about 300 years B.C.), wrote many works of considerable merit, acquired great proficiency in Sublime Geometry, and is said to have been the instructor of Euclid.

Euclid, according to Pappus and Proclus, was born at Alexandria,* in Egypt, where he flourished and taught Mathematics with great applause, under the reign of Ptolemy Lagus, (about 280 years B.C.) Some Arabian historians, however, inform us, that he was born at Tyre, that his father's name was Nancrates, an inhabitant of Damas. The particular place of his nativity appears, therefore, uncertain, but whether or not Alexandria had the honour of giving him birth, all historians agree that he flourished there as a teacher at the time above mentioned—which city, from that period till the time of its conquest by the Saracens, seems to have been the residence, if not the birth place, of all the most eminent mathematicians. It is supposed that Euclid studied at one time under the disciples of Plato at Athens. History is silent as to the time of his death. Pappus represents him as a person of great courtesy, mildness, modesty, and benevo-

* A town built by Alexander the Great, near the mouth of the Nile, in Egypt.

lence. He left various profound treatises on the most abstruse subjects belonging to his profession, but his eminent work is his Elements of Geometry, which supplanted all of a similar kind that preceded it, and such was the judgment he displayed in its composition, that notwithstanding the great additions made to Geometry since his time, it has continued for upwards of 2000 years to sustain the highest reputation as an elementary treatise.*

The great reputation of Euclid arose from the excellence of his Elements. His merit in this performance consisted in his having systematised the scattered discoveries of Thales, Pythagoras, Plato, and the lesser geometers who preceded him. No doubt he supplied important links. But when he infused coherence into the fugitive inventions of his predecessors, he imparted to Geometry the importance of a science.

Next after Euclid came Archimedes, who was born in the Island of Sicily. He established the true principles of mechanics, laid the foundation of Hydrostatics,† traced out the first rudiments of Naval Architecture, and applied his talents with such success to the cultivation of Geometry, Arithmetic, and Optics, that he has been styled the Newton of antiquity. He is said to have prosecuted his studies to the neglect of both food and sleep.‡ He perished

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* The curious may be interested to learn, that the Elements of Euclid were first published in *Latin*, in folio, by Radholt, at Venice, in 1482. First published in the original *Greek* by Hervagius, folio, Basil, 1533. And the first *English* translation was by Billingsley, folio, London, 1570, with a curious preface by John Dee.

† The science of weighing fluids.

‡ No great proof this of his 'philosophy.' Nothing is philosophical which cannot with prudence be imitated.

at the fall of Syracuse. A soldier came suddenly upon him in the museum, when he was intently engaged on some geometrical problem, and commanded him to follow him to Marcellus, (the Roman Consul who had taken the place). He refused, however, to stir till he had finished the subject on which he was engaged, and the soldier ran Archimedes through with the sword. When dying he desired that a sphere inscribed in a cylinder might be engraved on his tomb to perpetuate the memory of his most brilliant discovery.* The Sicilians, however, soon forgot him, and his tomb was found 200 years after by Cicero, by means of the symbol just named, in a field near Syracuse, overgrown with thorns. He was the most inventive, and the greatest Mechanical Engineer of ancient time.

Eratosthenes, a native of Cyrene, was called to the Alexandrian school by Ptolemy Euergetes, who made him his librarian on account of his extensive and general attainments in learning. His knowledge of Geography, Astronomy, and Geometry, enabled him to discover, for the first time on record, a method of measuring the circumference of the earth.

Fifty years after the death of Archimedes, Apollonius appeared, who gave a great impulse to the mathematical sciences, and whose discoveries were so highly esteemed that he was honoured by the appellation of the Great Geometer.

From the time of this eminent man we move on for three or four hundred years without meeting with one person who contributed to the advancement of the sciences.

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* He discovered some very fine proportions which exist between the sphere and the cylinder.

Then came Theon, (about 380 years, A.D.*) who, by his skill and perseverance in Mathematics and Philosophy, obtained the honourable dignity of being appointed president of the famous Alexandrian School, where, by his condition and conduct, he gained the greatest respect and reputation.

Theon had a remarkable pupil in his own daughter, Hypatia—a lady learned as well as beautiful. She was born at Alexandria. Hypatia was mistress of all the ordinary accomplishments of her sex, familiar with the most abstruse sciences, and made such progress in Philosophy, Geometry, Astronomy, and the Mathematics generally, that she was held as the most learned person of her time. She published commentaries on Apollonius's Conics, on Diophantus's Arithmetic,† and other works. She was chosen whilst very young to succeed her father in the same school, and to discourse from the presidential chair, at a time when Ammonius, Hierocles, and many other very learned men abounded at Alexandria and in various parts of the Roman Empire. Among her pupils, not less eminent than numerous, was the much esteemed Synesius, afterwards Bishop of Ptolemais. But it was not Synesius only and the disciples of the Alexandrian School who admired Hypatia for her virtues and learning. She was held as an oracle by the public, and was consulted by the magistrates on all important occasions. In short, when Nicephorus intended to pass the highest compliment on Eudocia, he thought he could not do it better than by calling her

* (380 A.D.)—380 years after Christ: A.D. being the abbreviation of *Anno Domini*, the year of our Lord.

† 'Diophantus was as great in Arithmetic as Archimedes, Apollonius, or Euclid was in Geometry, and by him was found out that admirable art we call Algebra.'—*Tacquet.*

another *Hypatia.* Whilst Hypatia thus reigned the most brilliant ornament of her sex in the annals of history, she was greatly admired by Orestes, the governor of the city, who, on account of her wisdom, often consulted her. This, together with an aversion which St. Cyril had against Orestes, proved the cause of her ruin. About 500 monks assembled, attacked the governor one day, and would have killed him had he not been rescued by the townsmen—and the respect which Orestes had for Hypatia, causing her to be traduced among the Christian multitude, they dragged her from her chair, tore her in pieces, and burned her limbs.* This shocking and brutal catastrophe was perpetrated in the lent of the year 416, in the time of Theodosius II.†

Pappus, a consummate mathematician, flourished about the end of the 4th century. His work, entitled 'Mathematical Collections,' has transmitted his name to posterity with distinguished lustre.

Proclus, who was at the head of the Platonic School, established at Athens (A.D. 500,) rendered important service to the sciences, and showed great kindness to all who embraced their pursuit.

The age of important discovery was past before the time of Pappus—and science in his time was beginning to decline. The labours of Proclus and a few of his followers were the last expiring efforts

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* It is related by Damasius and Suidas, and clearly proved by such learned men as Bruker, La Croze, and Basnaoe, that St. Cyril incensed the Christian populace against her.—(*Voltaire's* Phil. Dict. art. Hypatia.) St. Cyril was a man of parts, and there is a difficulty in accounting for the inhuman principles that actuated him in this affair.

† For a more particular account of this illustrious victim of fanaticism, see Bossut's Hist. Mat., English Ed., 8vo., 1803.

1st Ed

made in Greece for ancient Geometry. The Alexandrian School still existed, and science might have continued to flourish, had not the conquest of Egypt by the Arabians, who, with the design of spreading their religion and their empire, inundated Christendom in the seventh century, and not only gave it a fatal blow in Alexandria, but also extinguished its [redacted] languid existence in Greece.

Throughout all the civilised parts of the world in those times, the Alexandrian Library was known as the treasure-house of literature and knowledge. But neither the members nor the materials of this sacred edifice were respected by the Arabians, who, stimulated perhaps more by religious fanaticism than barbarous ignorance, sacked Alexandria, and massacred the philosophers who were assembled there for the purposes of study from all the neighbouring countries. Their laboratories and instruments were destroyed, and finally the Library itself, the accumulated scientific treasure of ages, containing above 200,000 written volumes, was committed to the flames for the mean purpose of heating the public baths. Caliph Omar, as he gave orders for the annihilation of the books, justified it by the strange, but convenient argument, that—'if the volumes were conformable with the Koran, they were superfluous; and if contrary to it, impious.' With this brutal act of bigotry and devastation may be said to have closed the scientific career of antiquity.

As a partial amends for so odious an outrage against civilization, the Arabians, about a century after, began to cherish the sciences of Geometry and Astronomy, and thus afforded an asylum which preserved them from entire extinction. About the time of their learned prince Almamon, several of their most celebrated works on Geometry were translated into the Arabic. Geber Ben Aphla laid the foundation of the

modern Trigonometry. An elegant treatise on Mensuration was written by Bagdadin; and it appears from his treatise on Optics, that Alharin was a superior geometer. They studied also the more sublime Geometry, improved Trigonometry, and reduced it to a convenient form.

Of the few Persian geometers, Nassir-Edin and Maimon-Reschid were the most distinguished. They both composed commentaries on Euclid. Persian learned men called Geometry the difficult science, and did not make any improvements in it—neither did the Turks nor Hebrews, although they translated some of the elementary works.

In one of the ancient treatises on Astronomy of the Hindoos, called Suryá Sidhánta, there is a correct system of Trigonometry; and hence it is concluded that the Indians had cultivated Geometry before the composition of this treatise. The Chinese were acquainted with the 47th of the First Book of Euclid.

The Romans knew little more of Geometry than the practice of land measuring. Varro, however, composed a treatise on Geometry, and Cicero had some acquaintance and esteem for it too. Vitruvius and Boetius had some knowledge of Geometry, but the latter is better known for his '*Consolations of Philosophy*.'

The learned Bede, who lived at the beginning of the 8th century, was acquainted with all the branches of the Mathematics. Under him Alcuin studied, who was well versed in mathematics, and was preceptor to the celebrated Charlemagne. He was a native of Britain, where, about this time, more attention was given to the study of this science than in any country in Europe.

For nearly two centuries after this period, the study of the science was extinct. No mathematician

of any eminence appeared till the beginning of the 15th century, when, with the general revival of learning, this science was destined to commence a new and more splendid career.

The Italians, by their intercourse with Arabia, became, about the end of the 12th century, the restorers to Christendom of its long-forgotten arts and sciences. Leonardus Pisanus, a rich merchant of Pisa, who had made, in the course of his profession, several voyages to the East, is said to have been the first who disseminated amongst Europeans a knowledge and a love of mathematical studies, after both had disappeared for so many ages.*

About the beginning of the 13th century, Campanus, of Navarre, translated Euclid, and several geometrical works were translated from Greek and Arabic MSS., by Maurolycus, of Messina, who was distinguished for some original and elegant works of his own. Ramus, or La Ramée, an original metaphysical writer, also composed several excellent works on Geometry: and Willibrad Snellius published at the age of seventeen an attempted restoration of an ancient work of much value.

About the beginning of the 16th century the Spanish and Portuguese geometers, John of Royas and Nonnius or Numer, lived. The former was the inventor of the projection of the sphere, and the latter was the first to introduce the Arabic System of Algebra into Europe. About the same time, Wright, a skilful geometrician, invented his chart—improperly called Mercator's chart. At this time, Germany produced the three mathematicians, Werner, Rheticus, and Pitis-

* A MS., dated 1202, of this celebrated Italian, has lately been discovered by Cossali, a canon of Parma.

ens. Werner made a German translation of Euclid. The Italian geometer, Lucus Valerius, determined the centre of gravity in various mathematical figures, conoids and spheroids.

Kepler was the first to conceive the theory of magnitude being composed of indefinitely small elements of the same kind; as, for instance, that a circle is composed of the surfaces of an indefinite number of very small triangles having the centre for their common vertex, and their small bases on the circumference. By means of this theory, though not very satisfactory in regard to scientific rigour, he extended Geometry considerably beyond the stage in which it was left by Archimedes. Cavallerius improved on the theory of Kepler, and effected a still further extension of Geometry by conceiving lines to be composed of points, surfaces of lines, and solids of surfaces.

Galileo, so eminent for his discoveries in physical science, did little to improve Geometry. He invented the Cycloid, called the Helm of Geometry on account of the discussions that originated concerning its properties. Gregory St. Vincent and Andrew Tacquet were two Flemish mathematicians, the former prosecuted the quadrature of the circle—the latter, the quadrature of surfaces and conic sections, with considerable success. He rendered essential service to the study of the science by subjoining the *Uses* to the propositions in an edition of Euclid's Elements, he published, after the manner of the celebrated French Jesuit, Claude François Milliett Dechales. Blaise Pascal, a Frenchman, (born 1623, died 1662) was equally distinguished for his great literary and scientific attainments, was the author of several valuable researches respecting the properties of the Cycloid; and at the age of sixteen, he composed a complete and elegant treatise of conic sections. But his health was

so much impaired, that soon after he was compelled to suspend his studies for four years; and he ultimately perished at the age of 39, after a lingering illness, in Paris.

Descartes, another eminent Frenchman, (born 1596, died 1650), had a profound knowledge both of mathematical and metaphysical science, was one of the most efficient agents in accomplishing that intellectual revolution which was in progress during his time, and was the inventor of a method which produced a thorough and memorable improvement in Geometry. Before his time the application of Algebra to Geometry, in the ordinary sense of the term, was known, as examples of it are to be found in the works of some ancient geometers. But Descartes invented that more general mode of applying it—that the science of Geometry, instead of being confined within the narrow limits imposed by the complexity and inadequacy of the ancient methods, is now of indefinite extent.

Hüygens, a native of Holland, (1629—1705) was a profound mathematician. He solved a problem of such difficulty, that it had not been previously attempted.

The year 1642, which 'gathered' Galileo 'to his fathers,' gave to the world Isaac Newton, the Archimedes of modern times, or, as he has sometimes been styled, from the number and importance of his discoveries, the Prince of Modern Mathematicians. The fame of Sir Isaac rests chiefly upon the *application* he made of his acquisitions, in explaining the laws or order by which the mechanism of the universe appeared to him to be regulated. His views with regard to this matter have been styled, in honour of him, the Newtonian System of Philosophy, and have been received implicitly by the learned world till nearly the present time.

The impetus given to mathematical pursuits by

the genius of Newton, has descended to our day, to use a phrase of his own, 'with increasing momentum.' For in his train have followed the illustrious names of Barrow, Keel, Gregory, Bishop Horsley (editor of an edition of Euclid's Elements), the Rev. Mr. Lawson, Robison, and many others. Among the most celebrated, rank Leibnitz, Euler,* the Bernouillis, Clairant, D'Alembert,† Thomas Simpson, Robert Simson, the well known restorer, and Playfair, the improver, of Euclid—Young, Leslie, and Madame Agnesi,‡ and Baron Swedenborg.§

It is not necessary to extend this enumeration. The mathematicians and geometers of this generation are known to the public. But it is above estimate how the diffusion of knowledge, one moment dreaded as mischievous, and in the next with the customary inconsistency of folly, pronounced impracticable, will increase the public taste for this science, and create a just and enduring appreciation of this useful order of scholars.

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* Euler lost his sight by intense application, and like Milton, closed his labours in blindness.

† One of the writers in the celebrated French 'Encyclopædia.'

'‡ This lady deserves the most respectful mention. Her Algebraical Works, in quarto, translated by Colson, have been long before the public, and very deservedly esteemed.'—*Harris's* 'Compendium of Algebra.'

§ Swedenborg's 'Principia,' a work declared, only very lately by one of the highest professors in Europe, 'not unworthy of being placed by the side of Newton's.'—*Douglas Jerrold's Mag.*, vol. 2, page 91.

CHAPTER IV.

UTILITY OF MATHEMATICS AS A MEANS OF EXTENDING OUR KNOWLEDGE OF THE PHYSICAL WORLD.

Plato denied the world can be
Governed without Geometry.
Butler's Hudibras.

It is a very trite observation, that human knowledge has been greatly extended by means of this science. Besides the important properties of magnitudes, and wonderful relations of abstract quantities which it has made known, it has unfolded a very extensive range of natural phenomena. It has investigated the principles of theoretical mechanics, the laws of equilibrium and motion of fluids, fixed and elastic. Optics, electricity, and magnetism are its debtors. The theory of acoustics, the propagation of light, and numerous other branches of science, have received great improvements by its agency. Even its reputed conceits have been susceptible of useful application. The ancient doctrine of the Conic Sections, which for 2000 years was an object of mere curious speculation, became, in the hands of Newton, a most efficient means of unfolding the planetary motions.

'Without the aid of rules derived from this science, the navigator, relying on his compass as a guide, could not with safety venture to any considerable distance on his element; intercourse with transmarine regions would be impossible; and, consequently, our knowledge of the globe we inhabit would be very limited. We should probably yet believe that its surface is an extended plane, and that it is supported on pillars; or, as was the opinion of some ancient philosophers, that

its figure is cylindrical, like a drum. Without the aid of this science, our knowledge of celestial bodies would be still more imperfect, and the consequences of our ignorance still more striking. We should still believe that these objects are equally distant from us, and, very probably, that they are distributed on the surface of an extensive crystalline sphere, performing a diurnal rotation about the earth, as the centre of the universe. We should also believe that some celestial phenomena, as eclipses and comets, are certain signs of a conflict of the elements of nature, or that they are the portentous indications of the wrath of heaven, while contemplating to inflict on superstitious mortals some dire calamity, as war, pestilence, or famine.

'How different from these unsatisfactory and incoherent conjectures is that great achievement of this science—the clear and satisfactory exposition, on the most incontrovertible principles, of the complex though sublime and systematic mechanism of the heavens; by which the distances and magnitude of the sun and planets have been measured, and also their weights, and even that of their satellites, ascertained; and by which the masses and distances of some of the stars, or suns of other systems, though inconceivably remote, even in comparison with the great extent of our own system, will probably ere long be determined.'*

An abler pen has celebrated this theme. 'The scientific principles that men employ to obtain the foreknowledge of an eclipse, or of anything else relating to the motion of the heavenly bodies, are contained chiefly in that part of science which is called Trigonometry, or the properties of a triangle, which, when applied to the study of heavenly bodies, is called Astronomy; when applied to direct the course of a

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* Bell.

ship on the ocean, it is called Navigation; when applied to the construction of figures drawn by rule and compass, it is called Geometry; when applied to the construction of plans of edifices, it is called Architecture; when applied to the measurement of the surface of any portion of the earth, it is called Land-surveying. In fine, it is the *soul* of science. It is an eternal truth. It contains the mathematical demonstration of which man speaks, and the extent of its uses is unknown.'*

The author of the Geometrical Companion defends the practical uses of Geometry with the ardour of an enthusiast. His examples are happy and forcible.

'A bee, which is an instinctive geometer, finds the use of it in constructing its comb, so as to be the most roomy and durable at the least expense of time and labour. Will any one after this fact doubt the use of Geometry to an intelligent creature?

'Almost all the evolutions of an army consist in converting one parallelogram into another. And the works of a fortified town, or position, are little else than combinations of parallelograms forming bastions, curtains, and parapets.'†

In our 'Uses,' other applications of this science will be seen. Perspective, Dialling, and a variety of the arts, might further be adduced. Literature gathers from it many of its prettiest similes. Dr. Johnson has two examples in his Life of Addison:—'A simile may be compared to *two lines converging to a point*, and it is more excellent as they approach from a greater distance.'

'An exemplification resembles two *parallel lines*, which run on together without approximation, never far separated, and yet never joined.'

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* Paine. † Darley.

Emerson has a fine thought adorned with the vestments of geometry. 'Every natural process is but a version of a moral sentence. The moral law lies at the centre of nature, and radiates to the circumference.'

CHAPTER V.

MATHEMATICS AS A MEANS OF MENTAL DISCIPLINE.

The same excellent persons in the commonwealth of learning who discovered Philosophy discovered also the Mathematics: like twin sisters they came into the world, and like twin sisters there is a natural affinity between them.—*Tacquet.*

It is a mortifying reflection that the soberest of sciences should have been disfigured by the exaggerations of untutored enthusiasm. The Mathematics have been extolled as the only source of logical acumen and cool judgment. Its discipline has been assumed to be the most salutary to which the human powers can be subjected. Some indiscreet votaries have carried its principles out of their legitimate province, and applied them to subjects they were never fitted to investigate. As might be expected, these proceedings have led to the disappointment of those who believed such unwarranted pretensions, and to the ultimate undervaluing of the real and substantial merits of Mathematics. No more than the varying constitutions of mankind admit of one panacea for human ailments, do the variations in human intelligence admit the pretensions of one science to train up the faculties of every individual. Natural Philosophy, Political Economy, Morals, Metaphysics, each has its peculiar discipline by which it addresses peculiar persons, and matures

their powers where Mathematics would entirely fail. The discreet friends of Mathematics never hesitate to allow this, and with the judicious discrimination of Tacquet, care only to claim for it kindred affinity to Philosophy, and the moderate but just honour of its having shed a certain lustre over human manners, and the arts and the sciences at large.

Dr. Arnott, author of the Elements of Physics, has attacked the assumed pretensions of Mathematics, and though he is far from doing it skilfully, it cannot be denied that he had occasion for his reflections. 'There is,' he says, 'no occupation which so much strengthens and quickens the judgment as the study of *Natural Philosophy*. This praise has usually been bestowed on Mathematics; yet a knowledge of abstract Mathematics existed with all the *absurdities of the dark ages*; but a familiarity with Natural Philosophy, which comprehends Mathematics, and gives tangible and pleasing illustrations of its abstract truths, seems incompatible with any gross absurdity. A man whose mental faculties have been sharpened by acquaintance with these exact sciences, in their combination, and who has been engaged, therefore, in contemplating *real relations*, is more likely to discover truth in other questions, and can better defend himself against sophistry of every kind. We cannot have clearer evidence of this, than in the history of the sciences, since the Baconian method of *reasoning by induction* took the place of the visionary *hypotheses* of preceding times.'

Dr. Arnott does not seem an enemy to pretension, provided he is allowed to advance it. The kind of eminence he denies to Mathematics, and which might safely be denied of any science, he does not hesitate to claim for Natural Philosophy. He unhesitatingly asserts that 'there is *no occupation* which so much strengthens

the judgment as the study of Natural Philosophy.' But common experience, in the variety of intellectual aptitudes, teaches us the fallacy of this broad assertion. It is not a disparagement, as Dr. Arnott supposes, that 'Mathematics existed with all the absurdities of the dark ages;' on the contrary, it is no little praise that Mathematics had vitality enough to exist at all under that weight of superstition and barbarism, which buried all other learning in undistinguished ruin.

A geometrician, whose ingenious performances I shall have several occasions to cite, and whose popular pen will give him considerable influence in other quarters, has contributed a eulogium to his favourite science, which cannot be permitted to pass unnoticed.

'Geometry not only sharpens our faculties, corrects precipitancy, and prevents liability to imposition, either from the arguments of others or our own, but it gives us a satisfaction in our knowledge [which we could not otherwise obtain], and thereby enables us to speak with a strength and perspicuity which a vague persuasion of the truth of what we utter could never inspire.'

Mr. Darley affirms too much. He plainly alludes to 'satisfaction' in knowledge in general, and the words I have put in brackets should have been omitted, as men can, for thousands do, 'otherwise' succeed in securing that satisfaction.* It is enough to contend, which

* It is but fair to this writer to state that he has deduced a compliment to Geometry from the biography of distinguished men, expressed with perfect propriety:—

'Alexander, Cæsar, Charles 12th, Buonaparte, and two of the most celebrated military engineers, Vauban and Coehorn, were profound geometers. And it is to their knowledge of this science, the superior excellence of their systems must be attributed.'—*Geometrical Companion.*

would be true, that the study of Geometry is *one* of the modes of educational discipline, and a most approved method indeed. It is contending that it is the only method which discovers vanity and offends good taste.

When a stranger, an enemy, or an indifferent person, chooses to laud Mathematics to the disparagement of other branches of knowledge, the encomium may be tolerated, though by no means imitated. Dugald Stewart, who had no particular predilection for the Mathematics, thus speaks of its study in his Philosophy of the Human Mind :—

'The intellectual habits of the metaphysician afford little or no exercise to that species of attention which enables us to follow long processes of reasoning, and to keep in view all the various steps of an investigation till we arrive at the conclusion. Such processes are much longer in Mathematics than in any other science; and hence the study of it is peculiarly calculated to strengthen the power of steady and concatenated thinking ; a power which, in all the pursuits of life, whether speculative or active, is one of the most valuable endowments we can possess.'

The Geologist, the Astronomer, the Chemist, severally have lauded their respective sciences as the most important that can engage the attention of mankind, both in an absolute and educational sense. This is the language of ignorance or vanity, Mathematics need never descend to the disparagement of other sciences. It has its distinctive merits: it can bear any comparison, its province is peculiar, its utility unquestioned, and its method of procedure instructive. These facts are its proper eulogics, and time has failed to obscure, or the envy of ages to diminish, their lustre.

Some of the features of discipline which indisput-

ably characterise mathematical study, have been well depicted by a modern writer.

'In reasoning, as in other arts, we are not masters of what we do, till we do it both well and unconsciously: now this advantage a judicious cultivation of Mathematics supplies. It familiarises the student with the usual forms of inference, till they find a ready passage through the mind, while anything which is fallacious and logically wrong, at once shocks his habits of thought, and is rejected. He is accustomed to a chain of deduction, where each link hangs on the preceding; and thus he learns continuity of attention and coherency of thought. His notice is steadily fixed on those circumstances only in the subject on which the demonstrativeness depends, and thus that mixture of various grounds of conviction which is so common to other men's minds is rigorously excluded from his. He knows that all depends upon his first principles, and flows inevitably from them; and that however far he may have travelled, he can at will go over any portion of his path, and satisfy himself that it is legitimate; and thus he acquires a just persuasion of the importance of principles on the one hand, and, on the other, of the necessary and constant identity of the conclusions legitimately deduced from them.'*

Dr. Olinthus Gregory, in his farewell address to the pupils of the Royal Military Academy, speaks of the study of Geometry as leading to that valued attainment—'*The power of mastering the mind.* If it be desirable to obtain and keep the ascendancy anywhere, it is, surely, *at home*, in the centre of your own intellect and its principles, your heart and its emotions.

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* Thoughts on Mathematics as a part of Liberal Education, by the Rev. William Whewell, Cambridge.

Gain the mastery here, govern within, learn to direct your thoughts to any subject you please, and keep them uninterruptedly to their occupation, till at *your* bidding the labour shall be remitted: then all will be well, for all will lead to a prosperous issue.'

We all know, that if human legs made peregrinations without the consent of the owner, how great would be the hindrance to business, yet how often is thought found roaming the world over when it should be confined at home But the command of the thoughts is as essential to success in thinking as the control of the limbs is to the dispatch of business. Inasmuch as Mathematics imparts this power, it is valuable discipline.

In the same address Dr. Gregory justly remarks, that the power of doing one thing at one time is the hardest and the most important lesson a young man has to learn. He is of opinion that the concentration of attention and good sense included in this habit, is powerfully impressed upon the notice of the mathematical student. The wisdom of this course, as applicable to children as well as adults, has lately been forcibly illustrated by a lady in these words:—'A very quick and clever child made an observation to her governess before me the other day, which had much of truth in it. "How is it, my dear," inquired the lady, "that you do not understand this simple thing?" "I do not know, indeed," she answered, with a perplexed look, "but I sometimes think I have so many things to learn that I have not time to understand."'

If we may trust the testimony of another enthusiast, this science exercises a species of influence of a very high order. 'The moral influence of the Mathematics did not escape the penetration of Plato, who, chiefly on that account, decides that the study

should be cultivated in his "Republic." "Geometry," says he, "is the knowledge of that which is eternal; it disposes the mind to the contemplation of truth, and gives a philosophical habit, which withdraws our thoughts from what is low to what is elevated. Let not the citizens of this renowned Republic neglect the study of Geometry."' Mr. Cooley concludes with observing—'We well know how much it conduces to the easy acquisition of any kind of knowledge, to have previously learned Geometry.'*

But it is far from my intention to rest the reputation of geometrical studies upon declamation, however eloquent, or upon testimony however respectable. An exposition of the Logic of Euclid is the best demonstration of the value of its discipline, and this I shall now attempt.

CHAPTER VI.

THE LOGIC OF EUCLID.

'Among the various guides which have been written for the direction of the understanding, no system of logic has ever been devised better calculated for this purpose than the Mathematics.'—*James Harris:* 'Introduction to Compendium of Algebra.'

MODERATING somewhat the absolute tone of Mr. Harris's opinion, it may be taken as expressive of the truth. The ancients considered Euclid their best book of logic. Its main features as such are, that from a few axioms, self-evident, and definitions, agreed upon, new truths are evolved by the perception of the *coherence*

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* Cooley's Introduction to the Study of Mathematics.

subsisting between them and the things previously admitted. These truths are added in their turn to the ranks of axioms from which others are to be inferred. Until this day sciences are considered perfect in proportion as they possess these features.* To Geometry belong three principal things:—Axioms, or first principles—Definitions, or explanations of the meanings of the chief terms employed—Postulates, or things required to be granted.

An enquirer must understand himself, and disputants must understand each other, or efforts after conviction will be futile. The geometrician (for we shall use him as the exponent of mathematical procedure) commences by distinguishing what he must assume: for such is the limited nature of human intellect, that all reason must primarily rest on assumption. Accordingly, Euclid lays down axioms which, being, as their name implies, self-evident, become the common point from which disputants can start on the path of investigation. In geometrical treatises, Definitions are usually placed first, but in logical propriety Axioms demand first to be considered, as the assent to definitions is often made by a secret reference to axioms in the thoughts.

AXIOMS.

The first principles, or fundamental axioms commonly used in Geometry are these:—'If a thing be divided into parts, the whole is greater than any one of the parts. Two straight lines cannot enclose a space. All lines drawn from the centre to the circum-

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* Mr. James Milne, in a curious work on National Education, has constructed outlines of Political Economy, Metaphysics, and other subjects, on these principles. Though not entirely successful, they are instructive speculations.

ference of a circle, are equal to each other. Two things cannot be both equal and unequal at the same time. Things equal to the same thing are equal to one another.'* The information conveyed by such elementary truisms is necessary: they are the very A, B, C, of the science—the lowest steps of the mathematical ladder, and as essentially useful as the highest.

The patient reference which the geometer makes to these first principles is highly instructive. He never loses sight of them. At his highest altitude he turns to refresh his understanding at those clear fountains of sense—an example which every reasoner may imitate, and an advantage which the investigator in every science may profitably secure to himself.

Dr. South has encouragingly remarked, that the reason of all things lies in a narrow compass.† The geometrician is professionally master of this secret, so true of every rational theme of human speculation. An Iliad of meaning lies concealed in the nut-shell of a fundamental axiom. Of what importance is this truth to the enquiring public of our day?

'How many readers are there who would not be glad of attaining to knowledge the shortest way, seeing the orb thereof is swollen to such a magnitude,

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* It strikingly evidences the intimate connection between Logic and Geometry that this axiom and its converse are found at the root of Aristotle's Syllogisms. Syllogisms were deducible from these two principles—First: things which agree with the same thing, agree with one another. Second: things whereof one does, and one does not agree with the same thing, do not agree with one another.—*Bruce.*

† The reason of things lies in a narrow compass. Most of the writings in the world are but illustration and rhetoric, of little consequence to a mind in pursuit after the philosophical truth.—*Dr. South.*

and life but such a span to grasp it? How many who have not some curiosity to know the *foundations* of those tenets upon which they so securely trust their understanding? or where the *footsteps* of those opinions and precedents may be found, which have given direction to so many modern performances? *

No science is fully mastered till the principles on which it is based are clearly seen. When these are lost sight of, all subsequent reasoning is confused. The geometrical student is early and much accustomed to keep these important considerations before him. The most abstruse proposition is an axiom to the mind that perceives its truth, and the most complicated science resolves itself into a very simple affair when its first principles become familiar to the understanding. These considerations reduce learning to an art, and give tone, power, and comprehensiveness to the understanding, which fit it peculiarly for the acquisition of knowledge.

Ordinary experience in learning and teaching must frequently impress the truth of these remarks on careful thinkers. When a pupil has a voluminous grammar put into his hands, he feels, with Dr. Johnson, that a great book is a great evil. But when the first principles of language are clearly seen, the difficulties vanish and the study is mastered.

This is not only the secret of the acquisition of knowledge, but also the agent for retaining it. Around these first principles, as around a standard, the thoughts naturally associate. Touch but a remote chord of any question, and it will vibrate to the central principles to which it has once been well attached—every relative impression owns a kindred connection, and the moment one is attacked, it, like a faithful sentinel, arouses a whole

* Oldys.

troop, who, marshalled and disciplined, bear down with their whole strength, and challenge the enemy.

DEFINITIONS.

That confusion in the varying meaning of terms, which reigns through most subjects that agitate mankind, is unknown in Geometry. The Definitions at the very threshold for ever fix the question of sense. Every leading term is defined. There are no words used in Geometry whose meanings are so much alike, that the ideas for which they stand may be confounded together. If there is any distinction between the meanings of two terms, it is a total and a complete distinction. Locke, in illustration of this point, remarks—'The idea of *two*, is as distinct from the idea of three, as the magnitude of the whole earth is from that of a mite; this is not so in other modes, in which it is not so easy, nor perhaps possible for us to distinguish between two approaching ideas, which yet are really different; for who will undertake to find a difference between the white of this paper, and that of the next degree to it?'

The meanings once fixed in Geometry are ever after adhered to: the same word is for ever used in the same sense. For instance, the term *right* signifies *straight*, and *straight* is its everlasting meaning in Geometry. In political philosophy, *right* is a well-known stumbling block, which perpetually overthrew the fine genius of Burke. 'Natural right,' 'inherent right,' 'moral right,' and 'political right,' are only a few of the rights which every now and then some puzzled politician will pronounce wrong. It perhaps is not possible to restrict popular phrases as we can scientific terms. But the geometrician, perceiving the advantages of doing so, acquires the taste of precision, and takes care (which is practicable with

everybody) to define the *principal* terms he may employ in any given dissertation.

POSTULATES.

With that precision from which the geometer never departs, he defines his mode of procedure. For this purpose he puts up his Postulates or petitions to be permitted a certain latitude of action. This latitude is never exceeded. Thus neither his language nor his practice is left open to cavil. Admit my axioms, agree to my definitions, and grant my Postulates, says the independent geometer, and I will engage to demonstrate every proposition I lay down.

A proposition to be proved, or a problem to be solved, have common parts—the Proposition, Enunciation, Construction, Demonstration, Conclusion. (These parts, for the assistance of the uninitiated, are clearly distinguished in the following pages.) *

The proposition being plainly laid down, the enunciation fixes its meaning, the construction provides for its proof, and a rigid demonstration leads the way to the conclusion. Every step has its distinct reference, establishing its legality and fairness. There is no attempt at evasion—no manœuvre, no equivocation. This daring self-reliance cannot fail to impart manly qualities.

The student is distracted with no impatient attempts to accomplish two things at once. One thing at one time is the order of Geometry. In the language of the propositions, all circumlocution is avoided. What is intended is straightway expressed. No digressions

* One who does not understand the principles of Euclid's Demonstrations, knows absolutely *nothing* of Geometry: unless he attain this point, all his labour is utterly lost.—*Dr. Whately.* Preface to Logic,' p. xxi.

are endured. Nothing is admitted but that which has been sanctioned, or established, or is relevant. Nothing is asserted but that which is intended to be demonstrated. It is impossible that these methods of procedure can be perceived and practised by the student without his acquiring valuable lessons, applicable to other subjects, and to much of the conduct of life.

The reader, in this place, must be cautioned against the language in which the Logic of Euclid is sometimes eulogised. Here is an extract from a recent journal:—'Never does the mathematician say, such a proposition is true because it is probable; or, because the balance of the *pro* and *con* arguments preponderates in its favour; or, because history records it, and tradition affirms it; or, because the holy St. This declared it, and the learned Philosopher That believed it; or, because infallible *I* assert it. No! HE takes a nobler stand, HE claims more lofty ground; his words are—"I have actually and incontrovertibly PROVED so and so to be a fact; disbelieve me, *if you can*; refute me, *if you are able.*"' This is the folly that encourages the contempt with which this fine study is sometimes treated.*

The reason why the geometer neither refers to probability nor authority is, that the nature of his science enables him to do without them. In the affairs of mankind there are numerous vital questions, such as those of law and legislation, which, from the nature of things, must be decided by probability: by *pro* and *con* arguments, and such decisions are as respectable and as salutary, if not quite as safe, as the conclusions of Mathematics. Those speculations

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† It was this thoughtless arrogance which provoked the partial dictum ~~severe exclamation~~ from a recent critic—'This logical and mathematical school of literature can only hold its sway with narrow minds and in an unimpassioned age.'—*Douglas Jerrold's* 'Shilling Magazine,' vol. 1, p. 572.

—and others, curious and important ones, in which authority and personal conviction for a long time form the only accessible evidence—are not to be rejected on that account, but to such evidence is to be allowed its proper weight. Let not flippant arrogance, rejoicing in the strength of Geometry, depreciate the arguments peculiar and necessary to other subjects. The modest mathematician learns the lessons of valuing strict demonstration wherever it can be had, and rightly refuses conjecture where certainty is to be obtained. He knows, if he has general information, that every proposition cannot be proved after the manner of Geometry, but he has a right to expect that what is attempted to be demonstrated in the same way shall be demonstrated with equal strictness.

Geometry is theoretical and practical: demonstration, direct and indirect: and reasoning is synthetical and analytical. Grammar is an illustration of the synthetical; Chemistry of the analytical. In Grammar we begin with letters—these are combined into syllables — syllables into words — words into sentences—and sentences into discourses. This is synthesis, or putting together. In Geometry, the grand element of the whole superstructure is a point, a point becomes a line, a line a surface: and of these solids are composed. Geometry is synthetical. In Chemistry, water is decomposed and resolved into its elementary principles by means of analysis, or taking to pieces. This is a similar demonstration, though obtained by a method the reverse of the former. There is a third method—namely, Induction, which Hume, with more precision than Locke, calls the doctrine of proofs. Induction is inferring from several particular facts, or propositions, a general truth. All these modes have important uses in different investigations. Geometrical reasoning is inductive. The

Axioms and properties of Definitions are the facts from which first conclusions are deduced. Every proposition established, is an addition to these facts, which become the increased bases of inferences. From coherency between ideas, the geometrical logician proceeds to coherency between facts, and from the entire body of facts before him he discovers the coherency of his general inference. Reason is the ability to perceive coherences. Reasoning on the abstrusest theorems is nothing more than after having arrived, step by step, at a remote truth, discovering its connection with preceding facts in the same chain. Geometrical problems alarm the uninitiated, who are not aware that a little attention conquers the simple ones, and a little patience the most difficult. One in the secret thinks them all equally easy, because he sees they are all equally connected. So far as geometrical study affords practice in mastering these consecutive coherences, it may be considered as aiding the development of reasoning power.

The 'First Book of Euclid' is a perfect illustration of geometrical logic. It embraces the foundation of all the subsequent books. They deal with new propositions and advanced reasoning, but the principle of procedure is the same as in the 'First Book.' So when the inquirer understands that, he comprehends the principle of them all. This 'First Book' furnishes general principles of reasoning, which will carry the student, without other assistance, through every subject to which his attention can be called. He has only to use them as initiative guides—giving him a taste for strictness in every application—not as fettering him to one species of evidence and inference. The discretion here alluded to is that so well enforced by the author of *Hermes*. 'When Mathematics, instead of being applied to *exemplify* Logic, comes to supply its place,

no wonder if it pass into contempt. For when men, knowing nothing of that reasoning which is universal, come to attach themselves for years to *a single species*, a species involved in lines and numbers only, they grow insensibly to believe these last as inseparable from all reasoning, as the poor Indians thought every horseman to be inseparable from his horse.'

CHAPTER VII.

NATURAL GEOMETRY.

Mr. Macaulay, in his historical Essay on Lord Bacon, has some ingenious remarks on the natural origin of the species of Induction with which Bacon's name is associated. Similar opinions have been expressed respecting the Syllogism of Aristotle. One of the ablest opponents of John Locke has some profitable speculations of the same kind concerning reasoning in general, all bringing Science, Art, and Logic home to nature, and resting their foundation there. This is the most encouraging aspect of learning that can be presented to the notice of a pupil. No treatise of an educational nature should be without such a feature. If an inquirer once perceives that the germs of a desired art are familiar to him, he makes an important step in advance—he gains confidence. He finds that he has only to systematise and extend his actual knowledge. The barrier of exclusiveness is broken down, and he enters the common ground of the initiated. A geometrician before referred to, has furnished some interesting illustrations of this truth with respect to his favourite science, accompanied by not less useful remarks.

'Suppose Codrus died, and left his field, nearly a square, to be divided among his four sons, Caius, Gaius, Titius, and Tatius. A little consideration would show that it might be done by drawing lines from the middle of one side to the middle of the other, as A B, C D, (see fig. 1). Thus there would be four squares of one size for the brothers. The middle point would perhaps be found by *stepping* along one side, and *half* the number of steps back. Or with a line or a rod. Such modes would be practical, and such as would be suggested to the simplest mind—for the rudiments of Geometry must be in the human mind before the art itself could be formally known as a system.

'Suppose the field to be *three cornered* (see fig. 2). A little more (though not much) sagacity would perceive that its division would be effected by drawing a line from any corner D of the field to the middle point A of the opposite side—and from A to C and B, the middle points of the remaining sides.

'But if the field had been of the unmanageable form as fig. 3, something more than common judgment would be necessary to subdivide *this* into four equal portions. Something would be wanted beyond what the senses or uncultivated reason could furnish, to recognise the equality of the parts A, B, C, D, when the figure *had* been accidentally divided as required. Then men would begin to consider how they might determine the equality or non-equality of differently shaped figures. They would, as we are told was the practice of the early philosophers, draw figures in sand or dust to investigate their proportions: first, because it was useful, and afterwards, because curious. This would be land measurement in miniature. The next removal would be from the sand to the slate, from the dust to the board, and from these to parchment or paper. What before engaged peasants without doors, would now employ philosophers within: what was rude, vague, and unsatisfactory, would now become accurate, definite, and conclusive · what was done experimentally with the rod and the tape, would now be determined rationally with the rule and compass. Behold the gradual transformation of the practical into the abstract art—of primitive into refined Geometry!

Ask a *ploughman* how he manages to keep his furrows

parallel, and he will tell you, by driving his team and directing the share, so that each new furrow shall keep *at the same distance* from the last, throughout the whole length. In like manner the school boy rules his copy *straight* by keeping each end of the ruler at the same distance from the last line ruled. Thus is the *doctrine of parallels* found inherent in the commonest minds.

'Let us by all means get rid of the scholastic notion, that Geometry is a thing totally distinct from, and independent of, the material world—irrelevant to the ordinary business of life, and uncongenial with the common trains of human reflection: that it is, in fact, an ethereal system, not refined from the grossness of terrestrial conceptions, but absolutely free from all intermixture or affinity with what we see or know about earthly matters: and that it is drawn from the skies, as Socrates drew his moral philosophy, and is a study for those alone who agree to call each other "purely intellectual beings." No such thing. It is only the simplest and most certain of our ideas, gathered from practice and experience, rendered accurate by strict circumspection, and combined with elegant ingenuity into a system differing from others of human invention only in being more perfect. So far is it from being anything of a supernatural or occult science, that a man of strong judgment might, in a popular way, form to himself a system of Geometry almost without knowing how to *read*. In fact, a great many illiterate men *do so*. What a powerful system of natural Geometry must the celebrated Watt have organised in his mind! He was ignorant of this science (that is, of the written word of it), but he was as well acquainted with the common truths of it as the most deeply initiated mathematician, or he would never have made his great discoveries, which involve those properties.' *

A little reflection on the Elements of Geometry, as displayed in the following pages, will further show that the principles of the science have their rise in human experience. The 'cutting off the corner' of a street in walking evinces an intuitive knowledge of a

* Abridged from Darley's *Geometrical Companion*.

geometrical theorem. The elements are common to all comprehensions—Science develops and perfects them.

CHAPTER VIII.

PRACTICE AND THEORY—OR, THE DISTINCTION BETWEEN PRACTICAL GEOMETRY AND PURE MATHEMATICS.

PURE Geometry is theoretical. Euclid's Elements are considered pure Mathematics. In this species of Geometry everything is done in accordance with the Postulates of Euclid. No instruments, nor compass, nor rule, are used, and the results arrived at are all abstract and unapplied.

Practical Geometry, on the other hand, is seen in the constructions connected with the Propositions, and in the 'Uses' annexed to our 'First Book.' In Practical Geometry, compasses, sections, rules—all species of instruments are employed. It is the application of Geometry to the arts of life.

In pure Geometry something of the practical certainly appears in the 'Constructions' of certain of the Problems and Propositions, but is employed no farther than is necessary for the development of the demonstration—the practical is subordinate, while in Practical Geometry it is the essence. Pure Geometry chiefly demonstrates that a given Proposition is true, or that a given thing can be done. Practical Geometry, on the other hand, teaches all convenient methods of practice, and *applies* it to any or everything to which it can be useful. Such is the aptitude of human intelligence for reality, that when we assume to apply only the Postulates of Euclid, we think of the instruments; but this difference still remains—in Practical Geometry the rule and compass are in the *hands*, in pure Geometry only in the thoughts.

In this work it is attempted to exhibit the 'Beauties' of Euclid's Elements *theoretically*, and the 'Uses' *practically*—because where the theory only is exhibited the question is heard—'It is all very beautiful, but what is the USE of it?' If only the practice is shown, the remark arises—'I see it is so, but I do not see WHY.'* The question and the remark are both proper, and both should be met. These distinctions rule throughout Mathematics. Whatever chiefly pertains to demonstration, and shows that a certain thing *can* be done, or that a certain result *will* ensue, belongs to the province of pure Mathematics—while in Practical Mathematics attention is mainly devoted to modes of operation and to the application of principles. In the pure department, all is abstraction and theory—in the other, all application and practice.

CHAPTER IX.

EXORDIAL ADDRESS TO THE STUDENT.

MORE than 2,000 years have rolled away since the entire Elements were given to the world. A halo of interest, peculiar to no other work known among men, surrounds it. It arose in the morning of Science, flourished among the glorious Geometers of old, kept alive the sacred fire of Learning, when almost the whole earth was without one genial ray, and it now shines with undiminished lustre amid the splendour of modern discovery. It has resisted the attacks of countless critics, great and clever men—has supported the weight of innumerable commentators; still it remains the most perfect work of man, and the finest monument of his reasoning powers. It has been translated into all languages, has been taught for centuries in every mathe-

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* Ritchie's *Principles of Geometry*.

matical school of eminence, and is now reputed the best introduction to that science extant.

It may safely be asserted, that as Grammar is the key of literature, so is Geometry the key of the sciences. Without exaggeration, it has been contended that 'the only road to accurate and independent knowledge of most of the sciences lies through the study of Mathematics.'* Euclid is the great schoolmaster of the sciences. His Elements are the pass-word into their dominions. 'Without a sufficient knowledge of Mathematics, that great instrument of [all] exact inquiry, no man can ever make such advances in the higher departments of science as can entitle him to form an independent opinion on any subject within their range.'†

Amasis wondered at the skill of Thales when he measured the Pyramids by means of their shadow. Neither the power nor station of Amasis could place him on a level with the distinguished geometer. Ptolemy, another Egyptian king, is said to have asked Euclid if there was no easier mode of becoming a geometrician than by studying his Elements. 'There is no *royal road*,' was his memorable answer. The meaning of which is, that the acquisition of no single idea can be made in a *royal* way. Only one path was open to Ptolemy and to mankind. Science owns no idle votaries. The only condition on which it yields its secrets is that of attention.

Do not fall into the error of vanity, and neglect learning under the impression that it courts you. It does indeed invite you, but it is in the language of independence. It is a benefactor, and never descends to supplicate the acceptance of its favours. It bestows honour and power on those who seek its wealth, but it never yet conferred a gift on any but the earnest.

Revolve the last words of the venerable professor of Mathematics (Olinthus Gregory)—'In your mental and scientific pursuits; *define* to yourselves clearly the *purposes* which you have in view; see to it that they are in no way incompatible with the nature and the *duty* of man, and you cannot direct your aim at too high a point.'

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* *Review for the Many.* † Sir John Herschel.

THE FIRST BOOK OF EUCLID.

SECTION I.

DEFINITIONS.

1. A *Point* is that which has no parts, or which has no magnitude.*

2. A *Line* is length without breadth.†

Corollary.—The extremities of a line are *Points;* and the *intersection* of one line with another is also a *Point.* [Thus (fig. 4) the place A is a point].

3. *Right* or *Straight* lines are such that cannot coincide in any two points without coinciding altogether.‡ [This is substituted for Euclid's, that the Corollary may appear more evident].

Cor.—Hence two straight lines cannot enclose a space. Neither can two straight lines have a common segment—that is, they cannot coincide in part without coinciding altogether. [The 14th prop. will illustrate this].

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* A *Point* is that which has no parts.—*Dechales.*

A *Point* is that which has position, but not magnitude.—*Playfair.*

A *Point* is the extremity of a line having no dimensions of any kind—neither length, nor breadth, nor thickness.—*Geom. Lib. Useful Knowledge.*

A *Point* is a mark in magnitude, which is (supposed to be) indivisible.—*Zeno.*

A *Point* is the beginning, as it were, of all magnitude, as unity is of number.—*Tacquet.*

A *Point* occupies no space.—*Legendre.*

† A *Line* has length only, and wants all breadth, inasmuch as it is understood to be produced by the flowing of a point.—*Tacquet.*

‡ A *Right* line is that whose extremes hide all the rest.—*Plato.*

A *Right* line is the shortest of all those that can be drawn between two points.—*Archimedes.*

4. *Parallel* straight lines are such as are in the same plane, and which being produced ever so far both ways do not meet.* (See fig. 5).

5. A *Superficies*, or surface, is that which has only length and breadth.

Cor.—The extremities of a superficies are lines, and the intersections of one superficies with another are also lines.†

6. A *Plane*, or plane surface, is that in which any two points being taken, the straight line between them lies wholly in that superficies.‡

7. A *Plane Angle* is the inclination of two lines to one another, which meet together, but are not in the same right line.

[The two lines, which, meeting together, make an angle, are called the *legs* of that angle. A B, A C, (fig. 7) are the *legs* of the angle B A C. The point A, at which the legs meet, is called the vertex of the angle. An angle may be designated by a single letter, when its legs are the only lines which meet at its vertex. But when more than two lines meet at the same point, in order to avoid confusion *three* letters are employed to designate the angle at that point—the letter which marks the vertex of the angle being always placed in the *middle*. Thus (fig. 6) the lines G C and E C meeting together at C, make the angle G C E, or E C G: the lines G C, E C, are the *legs* of the angle, the

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* Lines are *Parallel* to each other if the perpendiculars be tween them are all equal to each other.—*Archimedes.*

Newton, in Lemma 22, Bk. I. of his 'Principia,' says that parallels are such lines as tend to a point infinitely distant.

† A *Superficies* has two dimensions, and is understood to be produced by the flowing of a line.—*Tacquet.*

‡ A *Plane* surface is a surface which is *perpetually even*, or *flat* throughout its whole extent.—*Darley.*

(On such *planes* all the figures in the First Book are supposed to be constructed).

A *Solid* is a magnitude having the three dimensions of space: length, breadth, and depth. Any one of the six sides of a *Cube* is a *surface*, or *superficies*—an edge is a *line*—a corner a *point*. (See fig. 30).

point C is the *vertex*. In like manner may be named the angle E C A, which is the sum of the angles G C E, G C A: and so of the other angles round the same point.

When the legs of an angle are produced (or continued) beyond the vertex, the angles made by them on both sides of the vertex are said to be *vertically* opposite to each other: thus, since G C is continued to D, and E C to F, the angles G C E, D C F, are said to be *vertically* opposite to each other. And so of the other angles on each side C.

It must be understood, that by an angle is not meant the *surface* between the lines which form it. For though the surface be increased by producing the legs, (to D and E, for instance, in fig. 7), the angle will still remain the same in magnitude. By an *angle*, in fact, is meant the *degree* of *width*, or *separation* between the lines that form it: thus, the *opening* between B A C is greater than the angle F A C.]

8. When a straight line, standing on another straight line, makes the adjacent angles equal to each other, each of these angles is called a *Right Angle*, and each of these lines is said to be at *right angles* to, or *perpendicular* to the other.* (See fig. 8).

9. An *Obtuse* angle is that which is greater than a right angle. (See angle C B A, fig. 49).

10. An *Acute* angle is that which is less than a right angle. (See angle A B D, fig. 49).

11. A *Figure* is that which is bounded by one or more lines. (See fig. 9). The space enclosed within a figure is called its *area*.

12. A *Circle* is a plane figure bounded by one line called the *circumference*, or periphery; of which all lines drawn from a certain point within the figure to the circumference are equal to one another.† (See fig. 10).

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* A carpenter's square is a 'right angle.' Pythagoras is said to have been the inventor of the Square.

A perpendicular line inclines neither to the right hand nor to the left. From this circumspect idea probably arose the precept of the poet—

'Far from extremes a *middle* course is best.'

† Till the time of Plato this was the only curve admitted into Geometry.

13. That point is called the *Centre* of the circle.

14. A *Diameter* of a circle is a straight line drawn through the centre, and terminating on both sides in the circumference.

15. A line drawn from the centre to the circumference of a circle is called the *Radius*, or *Semi-diameter*.

[*Radii* is the plural of radius].

16. *Rectilineal* figures are those contained by straight lines.

17. *Trilateral* figures, or *Triangles*, by three straight lines. (See fig. 11).

18. *Quadrilateral* figures by four straight lines. (See fig. 12).

19. Of three-sided figures: an *Equilateral* triangle is that which has three equal sides. (See triangle A C B, fig. 31).

20. An *Isosceles* triangle is that which has two equal sides. (See triangle A B C, fig. 37).

21. A *Right-angled* triangle is that which has a right angle.* See triangle A E D, fig. 136.)

[The angle opposite the base is called the vertical angle. A side of a triangle, considered in reference to the angle opposite, is said to *subtend* it, or be the *subtense* of the angle. When the side of any triangle is produced, as B C to D, (fig. 14) the angle A C D, made by the produced part C D, with the other leg A C, is called the *external* angle. And the angle A C B, adjacent to the external angle, is called the *internal adjacent* angle. The other two angles of the triangle are together called the *internal remote* angles: and of these, that of which the produced side is a leg, namely, A B C, is the *internal opposite*, and the other, namely, B A C, is the *internal alternate* angle.]

22. Of quadrilateral figures: a *parallelogram* is that of which the opposite sides are parallel. (See fig. 15).

23. The straight line which joins the opposite angles of a quadrilateral figure is called a *Diagonal*.

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* Any side of a rectilineal figure may be called the *Base*. In a right-angled triangle (see fig. 13) the side *a c*, opposite the right angle is called the *Hypotheneuse*, either of the other two sides *a b*, *a c*, the Base, and the one not taken the perpendicular.

24. The *Altitude* of a triangle or a parallelogram is a perpendicular drawn from the opposite angle or side, upon the base, as $a\ b$, $c\ d$. (See figs. 16 and 17.)

25. Of parallelograms: a *Square* is that which has all its sides and angles equal. (See fig. 97).

26. An *Oblong*, or *Rectangle* is a four-sided figure, that has equal angles but not equal sides. (See fig. 23). [When a four-sided figure has equal angles, they are always *right* angles. Every 'square' is a 'rectangle,' but every 'rectangle' is not a square.']

SECTION II

POSTULATES.

(Things required: from the Latin *postulo*, to require).

1. Let it be granted that a straight line may be drawn from any one point to any other point.

2. That a terminated straight line may be produced to any length in a straight line.

3. And that a circle may be described from any centre with any radius or interval.

SECTION III.

AXIOMS.

(Authorities, or things having authority: from a Greek word. An *Axiom* is a self-evident proposition).

1. Things which are equal to the same things, are equal to one another.

2. If equals be added to equals, the wholes are equal.

3. If equals be taken from equals, the remainders are equal.

4. If equals be added to unequals, the sums are unequal.

5. If equals be taken from unequals, the remainders are unequal.

6. Things which are doubles of the same thing, are equal to one another.

7. Things which are halves of the same thing, are equal to one another.

8. Magnitudes which coincide with one another, (that is to say, which fit together so exactly, that every point of the one lies on some point of the other,) are equal.

9. The whole is greater than its part.

10. All right angles are equal to one another. (Legendre demonstrates this).

11. Two straight lines which intersect each other cannot be both parallel to the same straight line.

[All these Axioms are not required in the First Book, but not being numerous they are inserted as samples of the *assumptions* of Geometry].

SECTION IV.

DEFINITIONS OF TERMS.

A *Theorem* is a truth which becomes evident by a train of reasoning called a *Demonstration*.

A *Problem* is a question proposed which requires a solution.

A *Lemma* is a subordinate or minor truth, which is established in order to be employed in the demonstration of a theorem, or in the solution of a problem.

A *Proposition* is a portion of science. It is a common name applied indifferently to theorems, problems, and lemmas. A proposition is a simple statement respecting any subject.—*Jas. Milne.*

A *Hypothesis* is a fact assumed without proof. (Thus, when it is affirmed that in an Isosceles triangle the angles at the base are equal, the *hypothesis* of the proposition is that the triangle is Isosceles, or that its legs are equal).

A *Construction* is the addition made to, or change made in, the original figure, by dividing or drawing lines, in order to adapt it to the argument of the demonstration, or the solution of the problem. The condition under which

these changes are made is as indisputable as those contained in the hypothesis. Thus, if a line be made equal to a given line, these two lines are said to be equal by *construction.*

A *Corollary* is a consequence easily deduced from one or more propositions.

A *Scholium* is a remark on one or more propositions, which explains the application, connection, limitation, extension, or other important circumstance belonging thereto.

A *Demonstration* is a process or reasoning. It is either *direct* or *indirect.*

[To *demonstrate* a proposition is to exhibit a sufficient reason for the action affirmed by that proposition.—*Jas. Milne.*]

A *Direct* demonstration is a regular process of reasoning, from the premises to the conclusion.

An *Indirect* demonstration establishes a proposition, by proving that any hypothesis contrary to it is contradictory, or absurd : and it is therefore sometimes called a *reductio ad absurdum.*

The *Data* or *Premises* of a proposition are the relations or conditions, granted or given, from which new relations are to be reasoned out, or a construction to be effected.

To *Bisect* is to divide into two equal parts.

To *Trisect,* to divide into three equal parts.

An *Enunciation* declares what is to be proved or done.

The *Conclusion* asserts that the thing required has been proved or done.

SECTION V.

EXPLANATIONS OF SIGNS.

The arguments employed to demonstrate any proposition in Geometry must be founded more or less on the antecedent Propositions, Corollaries, or on the Definitions, Postulates, Axioms, Hypotheses, and Constructions. These are respectively referred to, in the following pages, under the abridged forms *Prop., Cor., Def., Post., Ax., Hyp., Const.*

The text of Dr. Simpson, used in the Universities, did him infinite credit in his day : but language has advanced since,

and the amplifications and reiterations he employs are needed now only in oral discourses. Mr. Cooley appears to be the most successful of all modern editors of Euclid in adopting a style of superior brevity and clearness, and has thus effected an important abridgement without omitting a single step in the reasoning, or in the slightest degree impairing its strength and validity. His text is that chiefly followed in the succeeding section.

SECTION VI.

8 ### ~~THEOREMS AND PROBLEMS.~~

PRELIMINARY DIRECTIONS.

1. Remember that in Geometry all sections, paragraphs, and sentences are of nearly equal importance.

2. Read deliberately every paragraph, that you fully understand it.

> 'Learn to read *slowly*, all other graces
> Will follow in their proper places;'

is an all essential injunction to the young elocutionist, and equally applicable to the young geometrician.

3. Note carefully the *parts* of every proposition.

4. Make sure of the *true* meaning of every term you meet with.

5. Trace every reference.

6. Study each proposition until you can prove it yourself *without* the book—till the *principle* of the demonstration is familiarised to the understanding.

7. The Plates of diagrams are intended only to give initiative ideas, and not to obviate the necessity of making new figures. The learner will best understand that which he has made himself. Geometry is to be acquired best by practice. The student should in every case construct his own diagram according to the 'construction in each proposition, making it as large as convenient, as largeness of diagram assists the conception of its purport.

8. Students perpetually ask—Must the Definitions and Axioms be committed to memory? Not necessarily so. It matters little that the memory is taxed if the understanding is excited. Let the learner make the references to the Definitions and Axioms as often as they are mentioned—which will be at the precise moment

Printer's Copy.

PREFACE.

THIS work was originally prepared at the request of a Society which took great interest in popular education. Several things, however, interfered to prevent that body from publishing it, and it was returned to me with kind words of approval and regret. Since then, I have waited for an occasion of presenting it myself to the service and judgment of the public.

Possibly, this delay may prove an advantage to the book, since it has afforded me the opportunity of revising it, when the first fondness of authorship had subsided into the more solid ambition of writing for the improvement of others.

The criticism this confession may invoke, I shall not be found to deprecate, but shall keep in recollection the prudent advice of Helicane, and

> With patience bear
> Such griefs as I do lay upon myself.

My aims have been :—1. ~~To prescribe~~ Prescribing the limits of Mathematical learning, defining what must be acquired and what may be neglected. 2. The discussion of the supposed connection between Arithmetic and Mathematics, and explanation of certain important distinctions generally confined to the class room. 3. Presenting a history of the rise and progress of Mathematics, somewhat more complete than previous ones. 4. Extending the view of the utility of Mathematics as a means of gauging the physical world. 5. A de-

velopment of the argument in favour of this science as a means of mental discipline, guarded from the arrogance and exaggeration which have so often destroyed its efficiency. 6. An exposition of Mathematical logic, and the true principles of Science, illustrating the value of the one and the simplicity of the other. 7. The enforcement of Natural Geometry—resting it on the foundation of the common understanding, distinguishing its Beauties and Uses, adapting it to the wants of the many, to the capacities of the young, to the aptitudes of the uninitiated, and the exigencies of men of business—the people of little time and a definite purpose.

G. J. H.

☞ The word EUCLID, on the title page, contains the Diagrams of six of the most interesting and celebrated Propositions of Euclid. The number of each Proposition, and the number of the Book in which it is found, are placed at the foot of each letter. The Diagrams in E U C occurring in the First Book, the reader will discover, in the following pages, to what they relate. The Diagram in L is employed to prove that all the angles that can be made in a semicircle are right angles. The Diagram in I belongs to the Proposition respecting the proportion between triangles and parallelograms of the same altitude. The Diagram in D is the geometrical foundation of the Rule of Three. It seemed the most appropriate tribute to Euclid to associate his works with his name.

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INTRODUCTION TO THE MATHEMATICS.

THE FIRST BOOK OF EUCLID.

USES OF EUCLID.

INTRODUCTION TO THE MATHEMATICS.

CHAPTER I.

DISCOURAGING INFLUENCE OF A CERTAIN POPULAR MAXIM OVER THE PURSUIT OF LEARNING IN GENERAL, AND MATHEMATICS IN PARTICULAR.

'WHATEVER is worth while doing at all, it is worth while doing well,' is a maxim of deserved reputation: and *when* applied in its proper sense, of great service. But, adopted as this has been, without due regard to its exact import, it may be questioned whether any maxim ever proved so serious a discouragement to the majority of learners. What it is worth while to do at all, it is, unquestionably, sound wisdom and true economy to do well. But *what* it is worth while to perform, and the *measure* of the undertaking, are to be decided by the wants and circumstances of the person engaging in the pursuit. Instead of this being done we have those who, on feeling interested in the investigation of any branch of learning, conclude it to be incumbent on them to entirely explore it, whatever be its nature or extent, and, wanting time to accomplish this, often an extravagant task, remain in entire ignorance of that of which it was probably important they should know something.

The person destined for the profession of Chemistry, of necessity must acquaint himself with the works of Priestley, Davy, Dalton, Liebig, Faraday, Daniels, and other eminent men in that department of knowledge: but the general inquirer will sufficiently in-

form himself on this subject from Chambers' 'Rudiments' of the science. In like manner, a person intended for the Medical profession has a wide class of authors before him, with whom he must become conversant, while the artizan would be considered well informed who was familiar with the works of Southwood Smith and Andrew Combe. Now, *what* it is proper for the Chemist and Medical student to undertake, is no less than the whole range of knowledge belonging to each of their respective professions—and *this* it is worth their while to master well. But the general inquirer and the artizan, being differently circumstanced, require only such a general insight into these subjects as shall enable them to converse thereon with propriety, to understand books in which they are introduced, and to apply the plain and popular rules to the business, enjoyment, and preservation of life.

The judicious rule of acquisition is this. After mastering the knowledge proper to individual profession, we next have to seek general knowledge: for he who should essay to sound the depths of every subject, would find life exhausted before he had completed a tithe of his task. A course which would condemn to the unremitting toil of hoarding information, and exclude for ever from the pleasure of its application.

Unmindful of considerations of this kind, it is commonly thought, that whenever Geometry is proposed for investigation, (since whatever is worth while doing at all is worth while doing well), nothing less than Dr. Simpson's Eight Books of Euclid, his namesake's Trigonometry, Bonnycastle's Mensuration, Bridge's Algebra, and a host of scientific treatises, must necessarily be entered upon. And as it is only the time of professed students that will allow of this, the

The general enquirer determines what is properly worth his —

attention, and earnestly devotes himself to that, and to nothing else. His business is to *explore*, not to *conquer* the region of Mathematics It is that which it is worth his while to do, and he does *that* well.

Many people of undisciplined powers, and ignorant of the art of acquiring knowledge, can never move anywhere unless they move profoundly. Such people soon lose themselves. Instead of being the masters of knowledge, knowledge is the master of them. By being profound out of place, they bring profundity into contempt. It is not within the compass of human power to be profound upon more than a few subjects The poet was somewhat too strict in his limitation, but he approached the truth in declaring—

> One subject only will one genius fit,
> So vast is art—so narrow human wit.

Do not be deterred from the useful course of general investigation, in these two Sections recommended, by the vague charge of superficiality, so often preferred without propriety and listened to without advantage. Many persons live and die in disreputable ignorance because they are determined not to be superficial. Superficiality is a serious imputation when it respects what you ought to know well. But with respect to general knowledge, it is no more criminal to be superficial than to have only five senses, or not to be in half a dozen places at once. It is the condition of humanity only to be able to take a general survey of the wide field of knowledge. The republic of literature has no inapt resemblance to the state of good citizenship, in which a man is expected to know his own business, and to have such an acquaintance only with that of his neighbours as shall give him a laudable interest in their pursuits, and assure him that they are conducive to the dignity of the commonwealth.

troduce the reader to the History of the Mathematics, for which he is prepared. The record of its gra-

admirable Conversations on that subject, the requisite information—the First Book of Euclid will accomplish the rest. It will introduce him to both Theoretical and Practical Geometry, and to the technical characteristics of Mathematics. It will enable him to understand the *modus operandi* of Newton's Principia. It will familiarise him with the illustrations of Dynamics and Mechanics. After performing the 'Constructions' of Euclid, and putting them to the 'Uses' at the end of this book, any person could understand and practice Keith's principles of Mapping, and illustrate his geography by constructing his own maps. We have seen in the quotation in the last chapter, from the *Saturday Journal*, how the First Book of Euclid, if it does not make the young pupil master of Astronomy, it will place him on the same common ground with the Astronomer himself, and familiarise him with the scientific secrets of that fine pursuit. Indeed, if the First Book of Euclid does not make the student a Mathematician, it so qualifies him that he can be a Mathematician whenever he pleases, as it imparts a clear insight of that science which will enable him to

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Death of Zoilet

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CHAPTER II.

IMPORTANT DISTINCTIONS PERTAINING TO MATHEMATICS.

MATHEMATICS is the science of quantity and magnitude. It is usually defined as that which treats of whatever can be numbered or measured.* Mathematics embraces all modes of procedure, and includes all the arts and sciences employed in determining magnitude and quantity. Quantity is understood to include the distinctions of weight and measure. Magnitude is the consideration of size.

Geometry, though a very important science, is but a branch of the Mathematics. Geometry treats of the properties of lines, angles, surfaces and solids. The term Geometry is applied to Mensuration, to Trigonometry, Conic Sections, and to every department of investigation in which lines and angles, surfaces and solids, are chiefly concerned.

Arithmetic, the art of figures—and Algebra, the science of symbols, constituting the lower and higher departments of calculation, are both branches of Mathematics. But Geometry is the principal branch.

From these points of definition the relative distinctions subsisting among the branches of Mathematics, will easily be perceived, and the careful observer will be enabled to avoid the error of confounding these distinctions. The common mystery attached to the Mathematics has, in a great measure, its rise in this kind of inattention.

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*Ritchie's Principles of Geometry, page 2.

A person skilled in the use of figures is an Arithmetician. This is his proper appellation. If acquainted with that higher branch of calculation in which quantities are represented by letters, his correct designation is an Algebraist. The proficient in Euclid's Elements is a Geometrician. The more precise ancients always styled the illustrious author of the Elements, the Great Geometer. It is the person who has acquired skill in *all* these branches, and such others as pertain to the determination of magnitude and quantity, who is properly the Mathematician. As Mathematics includes all these arts, properly, the Mathematician is master of them all.

We often, indeed, hear the skilful Algebraist styled a Mathematician. The Trigonometrist and Geometrician often receive the same designation. But correct taste is never guilty of this confusion of terms. Such want of precision is a poor compliment to the precisest of sciences.

The term Mathematics will never be intentionally employed in this work, except in its comprehensive sense, as explained in its definition. That term on the title page is employed to indicate that an attempt will be made to lay bare its clear import, and by so doing divest it of its reputed mystery; and illustrating (as will be done in various places) the exact nature and influence of its branches, and afford a comprehensive and easy insight into its nature and application. But chiefly, this work, as its second title expresses, is confined to Euclid—to Geometry.

The Elements of Euclid are often designated the Elements of Mathematics. True, the Elements once constituted Mathematics themselves; but since the perfection of numerals and the invention of Algebra by Diophantus, they have sunk into a relative rank. It is true, Geometry gives laws to Arithmetic. The

dual rise out of the wants of men by patient study, and its progress, will further divest him of the prejudice, if any remain, of the popular mystery connected with this science.

CHAPTER III.

HISTORY OF THE RISE AND PROGRESS OF THE MATHEMATICS.*

'THE Mathematical Sciences were the first of all among men, if we may believe *Josephus*. He (Bk. I. Chap. 3,) writeth, that the posterity of Seth observed the order of the Heavens, and the courses of the Stars. And lest these inventions should slip out of the knowledge of men, *Adam* having predicted a two-fold destruction of the Earth, one by a Deluge, and the other by Fire, they raised two Columns, one of Bricks, the other of Stone, and inscribed their inventions upon them; and if the Brick one should happen to be destroyed by the Deluge, that of Stone, which would remain, might afford men an opportunity of being instructed, and present to their view the things which had been inscribed upon it. They also say, that that

* Compiled chiefly from Andrew Tacquet's (of the 'Society of Jesus') Elements of Euclid, translated by Whiston—the Introduction to the Elements of Geometry, translated into English from Peyzard, by Phillips—and from the History of the Science prefixed to the Elements of Plane Geometry, by Bell. But, of course, omitting the usually recited lists of the inventions of the 'Fathers of the Science.' If enumerated, who, on a general introduction to Mathematics, would feel enraptured or be enlightened by details of 'The Lair,' 'The Loci,' Pori, Quadratics, and Curves? They are best introduced where they are intelligible.

~~agents are lines and angles. It is a distinct,~~ entire, and independent science.

But it will be asked by the intelligent reader, if Mathematics comprises not only Arithmetic, Algebra, Differential and Integral Calculus, and Geometry—but also such principles as are employed in the measurement of the earth and the skies, in Physical Geography, Astronomy, Navigation, Architecture, and a variety of other branches, how is the maxim with which Sec. I. opens to be applied by the general enquirer to these departments? The answer is easy.

If the enquirer aims at the eminent distinction of being a Mathematician, he must prosecute each of these branches of study, so as to acquire familiarity with their modes of procedure, and some skill in their application. Nor will this be so difficult as at first sight appears. The nature of each is kindred, and Geometry is the key which will unlock them all—only it requires the application of many years to compass so wide a range of physical knowledge.

The course of the general enquirer is different. Arithmetic being required, to some extent, by every-

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for Arithmetic and several other arts. The Arithmetician and *Practical* Geometer, (a distinction hereafter to be explained) come to Euclid for the reasons of their rules. I am aware that Schiller (according to the *Dublin University Magazine*) makes the great distinction between art and pure science to consist in the element of humanity being always essentially involved in every work of art, and this element being excluded in all that is properly called science. But this distinction relates to high art—to the poetry of art. I have here to do with art in the sense of manipulation—and with science in the sense of systematized and demonstrative knowledge—and in every employment of these terms, allusion is made to the art and science of practice. No disparagement to Schiller's distinction, but ours is the other department.

body, *he* acquires its first principles, and as much of its practice as he is likely to need. If he is curious about Algebra, he may soon gather from Coles's admirable 'Conversations' on that subject, the requisite information—the First Book of Euclid will accomplish the rest. It will introduce him to both Theoretical and Practical Geometry, and to the technical characteristics of Mathematics. It will enable him to understand the *modus operandi* of Newton's Principia. It will familiarise him with the illustrations of Dynamics and Mechanics. After performing the 'Constructions of Euclid, and putting them to the 'Uses' at the end of this book, any person could understand and practice Keith's principles of Mapping, and illustrate his geography by constructing his own maps. We have seen in the quotation in the last chapter, from the *Saturday Journal,* how the First Book of Euclid, if it does not make the young pupil master of Astronomy, it will place him on the same common ground with the Astronomer himself, and familiarise him with the scientific secrets of that fine pursuit. Indeed, if the First Book of Euclid does not make the student a Mathematician, it so qualifies him that he can be a Mathematician whenever he pleases, as it imparts a clear insight of that science which will enable him to extend his studies whenever his duty or taste shall incline him. Thus the general enquirer applies our maxim. He determines what is properly worth *his* attention, and earnestly devotes himself to that, and to nothing else. His business is to *explore,* not to *conquer* the region of Mathematics It is that which ~~it is worth his while to do, and he does *that* well.~~

~~Many~~ People of undisciplined powers, and ignorant of the art of acquiring knowledge, can never move stet anywhere unless they move profoundly. Such people soon lose themselves. Instead of being the masters of

knowledge, knowledge is the master of them. ~~By being profound out of place, they bring profundity into contempt. It is not within the compass of human power to be profound upon more than a few subjects.~~ The poet was somewhat too strict in his limitation, but he approached the truth in declaring—

> One subject only will one genius fit,
> So vast is art—so narrow human wit.

Do not be deterred from the useful course of general investigation, in these two Sections recommended, by the vague charge of superficiality, so often preferred without propriety and listened to without advantage. Many persons live and die in disreputable ignorance because they are determined not to be superficial. Superficiality is a serious imputation when it respects what you ought to know well. But with respect to general knowledge, it is no more criminal to be superficial than to have only five senses, or not to be in half a dozen places at once. It is the condition of humanity only to be able to take a general survey of the wide field of knowledge. The republic of literature has no inapt resemblance to the state of good citizenship, in which a man is expected to know his own business, and to have such an acquaintance only with that of his neighbours as shall give him a laudable interest in their pursuits, and assure him that they are conducive to the dignity of the commonwealth.

~~Of the majority of the branches of learning, we can only hope to master the first principles. And it happens that no work that human wit has ever devised teaches the art of doing this in so striking a manner as does the First Book of Euclid. But more of this in the proper place. We must now introduce the reader to the History of the Mathematics, for which he is prepared. The record of its gra-~~

dual rise out of the wants of men by patient study, and its progress, will further divest him of the prejudice, if any remain, of the popular mystery connected with this science.

CHAPTER III.

HISTORY OF ~~THE RISE AND PROGRESS OF THE~~ MATHEMATICS.*

'THE Mathematical Sciences were the first of all among men, if we may believe *Josephus*. He (Bk. I. Chap. 3,) writeth, that the posterity of Seth observed the order of the Heavens, and the courses of the Stars. And lest these inventions should slip out of the knowledge of men, *Adam* having predicted a two-fold destruction of the Earth, one by a Deluge, and the other by Fire, they raised two Columns, one of Bricks, the other of Stone, and inscribed their inventions upon them; and if the Brick one should happen to be destroyed by the Deluge, that of Stone, which would remain, might afford men an opportunity of being instructed, and present to their view the things which had been inscribed upon it. They also say, that that

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Stone Pillar, which even in our day is seen in Syria, was dedicated by them. This *Josephus* says, whom I leave to vouch for the story.'*

Geometry must be coeval in its origin with the invention of the square and the compasses, and these being necessary to the rudest operations, were doubtless known in the earliest ages of the world. Geometry first assumed the character of a science, in Greece, about 2,500 years ago, and flourished during ten centuries: then was almost entirely neglected till about 200 years since, when it broke forth in Europe with more than its pristine brilliancy.

The term Geometry is derived from the Greek words *ge* (earth) and *metron* (measurement), literally signifying the *Measurement of the Earth*. But the science thus designated is now so extended, as to be equally applicable to the measurement of the heavens. Aristotle, the famous preceptor of Alexander the Great, ascribes its origin to the Egyptian Priests, and Herodotus, an ancient historian, to the time when Sesostris, who reigned in Egypt, (1500 B. C.)† intersected it by numerous canals, and apportioned the country among the inhabitants.

To Egypt appears chiefly due the honour of having been, if not the birth place, at least the cradle of Geometry, where it received its first culture. That old and mighty kingdom seems to have been the nurse, perhaps the mother, of almost all the useful arts and sciences. Hindoostan and China contend with Egypt for the pre-eminence with respect to Geometry; but by whichever it may have been first cultivated, it

* Tacquet's Historical Account of the Rise and Progress of Mathematics.

† B. C. Before the reputed Birth of Christ—before the commencement of what is denominated the Christian Era.

was certainly brought into general notice by the Egyptians.

Thales, the reputed father of Greek philosophy, born at Miletum (640 B. C.), is celebrated for being the first who introduced Geometry to Greece, and for the many highly interesting discoveries he made in it: for being able, by geometrical principles, to compute the distance of vessels from the shore, and for having measured* the altitude of the Pyramids of Egypt by means of their shadow, much to the astonishment of Amasis the king, who wondered at his scientific skill. From him Astronomy is said to have made considerable advances—and he founded an establishment in Greece known by the name of the Ionian School. It was in the age of this Philosopher, according to the best informants, that Geometry first began to assume the character of a science. He observed the Equinoxes and Solstices, and was the first who foretold the Eclipse of the Sun. And he foretold to King Cyrus an Eclipse of the Moon. The Electrical properties of amber were first remarked by him, upon which has been built the important science of Electricity.

The first elementary treatise on Geometry is said to have been composed by Anaximander, a disciple of Thales, and Thales's successor in his school.

Next in order comes Pythagoras,† the philosopher

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* Thales, one of the seven wise men of Greece, about 600 years before Christ, invented the following method for measuring the height of an Egyptian pyramid. He watched the progress of the sun till his body and shadow were of the same length, and at that instant measured the shadow of the pyramid, which consequently gave its height.—*Lord Kame's History of Man, vol.* 1, *p* 91.

† The supposed inventor of the term 'Philosophy'—signifying *a lover of knowledge.* A term said to have been first applied to Solon, an ancient law-giver.

of Samos, (an island in the Levant, between Greece and Asia), no less distinguished than Thales for the variety and extent of his discoveries. He is said to have travelled into Egypt and India in pursuit of knowledge, to have settled at Tarentum in Italy, (about 550 years B.C.), and to have maintained, in Astronomy, the true system of the world which places the sun in the centre with the planets revolving round it. From this philosopher it was called the Pythagorean System, and was revived two thousand years after his day by Copernicus. Pythagoras was distinguished in all his pursuits by a genius remarkably inventive and assiduous. He possesses no ordinary claim to the honour of posterity. As a mathematician, he was decidedly the first of his time,—as a philosopher, he delivered many curious things concerning God and the human soul, and a great variety of precepts relating to the conduct of life, both civil and political.

It is not certain at this time whether Geometry had been founded into a regular system. If not, considerable advances had been made, and ere a century had elapsed from the age of Pythagoras, Zendorus, a man of great parts, arose, whose writings are the first among the ancients which have survived the wreck of time, one geometrical tract of his having been preserved.

Then came Hippocrates, whose brilliant genius, great diligence, and application, rendered essential service to this science. He was the third who wrote on the Elements of Geometry, but on the appearance of the Elements of Euclid, his treatise was consigned to oblivion.

The origin of the celebrated Platonic School, (400 years B.C.), is considered one of the most important epochs in the history of the science. Plato, its

founder, who is called by way of pre-eminence the 'Athenian Sage,' and who was alike remarkable for eloquence, the poetical sublimity of his imagination, and the accuracy of his mathematical performances, gave to Geometry the form and substance of a complete science, and enriched it with many discoveries. Indeed, so profound a veneration did he entertain for it, that he made it a principal object of instruction among his scholars, and had written over the entrance of his academy, the well known words,—'*Let no one enter here who is ignorant of Geometry.*' Plato was the discoverer of the Conic Sections, (or those curves which are found on the surface of a Cone when cut through in different directions). He found out many of their most remarkable properties, which being made the continual study of his scholars and successors, at length became a distinct science from the common elements, and received the appellation of the *higher or sublime* Geometry.

Such was the versatility of his genius, that he directed his acute and original powers with eminent success to the cultivation of Moral Philosophy, and produced his famous work, entitled the 'Republic,' with the view of imparting to civilised intercourse the character of a science, and giving a philosophical direction to the ever jarring aims and interests of mankind.

The celebrated Aristotle, the successor of Plato, and preceptor to Alexander the Great, and who is justly regarded as one of the greatest men of olden time, made many improvements in the mathematical sciences. After attending the lectures of Plato, he opened a school himself in the Lyceum of Macedonia, which was assigned him by the magistrates, and was the founder (about 350 years B.C.), of the well known Peripatetic school.

Theophrastus, a disciple of Aristotle, composed a complete history of the origin and progress of Mathematics, Astronomy, and Arithmetic, from the earliest periods to his own time. The treatise has been unfortunately lost.

Archytas, of the academy of Plato and of Tarentum, the place in which Pythagoras settled, is famous as the constructor of a flying pigeon, and as the reputed inventor of the pulley and screw. 'He was one of the first,' says Tacquet, 'who brought down the Mathematics to human uses.' Aristœus, who flourished (about 300 years B.C.), wrote many works of considerable merit, acquired great proficiency in Sublime Geometry, and is said to have been the instructor of Euclid.

Euclid, according to Pappus and Proclus, was born at Alexandria,* in Egypt, where he flourished and taught Mathematics with great applause, under the reign of Ptolemy Lagus, (about 280 years B.C.) Some Arabian historians, however, inform us, that he was born at Tyre, that his father's name was Naucrates, an inhabitant of Damas. The particular place of his nativity appears, therefore, uncertain, but whether or not Alexandria had the honour of giving him birth, all historians agree that he flourished there as a teacher at the time above mentioned—which city, from that period till the time of its conquest by the Saracens, seems to have been the residence, if not the birth place, of all the most eminent mathematicians. It is supposed that Euclid studied at one time under the disciples of Plato at Athens. History is silent as to the time of his death. Pappus represents him as a person of great courtesy, mildness, modesty, and benevo-

* A town built by Alexander the Great, near the mouth of the Nile, in Egypt.

lence. He left various profound treatises on the most abstruse subjects belonging to his profession, but his eminent work is his Elements of Geometry, which supplanted all of a similar kind that preceded it, and such was the judgment he displayed in its composition, that notwithstanding the great additions made to Geometry since his time, it has continued for upwards of 2000 years to sustain the highest reputation as an elementary treatise.*

The great reputation of Euclid arose from the excellence of his Elements. His merit in this performance consisted in his having systematised the scattered discoveries of Thales, Pythagoras, Plato, and the lesser geometers who preceded him. No doubt he supplied important links. But when he infused coherence into the fugitive inventions of his predecessors, he imparted to Geometry the importance of a science.

Next after Euclid came Archimedes, who was born in the Island of Sicily. He established the true principles of mechanics, laid the foundation of Hydrostatics,† traced out the first rudiments of Naval Architecture, and applied his talents with such success to the cultivation of Geometry, Arithmetic, and Optics, that he has been styled the Newton of antiquity. He is said to have prosecuted his studies to the neglect of both food and sleep.‡ He perished

* The curious may be interested to learn, that the Elements of Euclid were first published in *Latin*, in folio, by Radholt, at Venice, in 1482. First published in the original *Greek* by Hervagius, folio, Basil, 1533. And the first *English* translation was by Billingsley, folio, London, 1570, with a curious preface by John Dee.

† The science of weighing fluids.

‡ No great proof this of his 'philosophy.' Nothing is philosophical which cannot with prudence be imitated.

at the fall of Syracuse. A soldier came suddenly upon him in the museum, when he was intently engaged on some geometrical problem, and commanded him to follow him to Marcellus, (the Roman Consul who had taken the place). He refused, however, to stir till he had finished the subject on which he was engaged, and the soldier ran Archimedes through with the sword. When dying he desired that a sphere inscribed in a cylinder might be engraved on his tomb to perpetuate the memory of his most brilliant discovery.* The Sicilians, however, soon forgot him, and his tomb was found 200 years after by Cicero, by means of the symbol just named, in a field near Syracuse, overgrown with thorns. He was the most inventive, and the greatest Mechanical Engineer of ancient time.

Eratosthenes, a native of Cyrene, was called to the Alexandrian school by Ptolemy Euergetes, who made him his librarian on account of his extensive and general attainments in learning. His knowledge of Geography, Astronomy, and Geometry, enabled him to discover, for the first time on record, a method of measuring the circumference of the earth.

Fifty years after the death of Archimedes, Apollonius appeared, who gave a great impulse to the mathematical sciences, and whose discoveries were so highly esteemed that he was honoured by the appellation of the Great Geometer.

From the time of this eminent man we move on for three or four hundred years without meeting with one person who contributed to the advancement of the sciences.

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* He discovered some very fine proportions which exist between the sphere and the cylinder.

Then came Theon, (about 380 years, A.D.*) who, by his skill and perseverance in Mathematics and Philosophy, obtained the honourable dignity of being appointed president of the famous Alexandrian School, where, by his condition and conduct, he gained the greatest respect and reputation.

Theon had a remarkable pupil in his own daughter, Hypatia—a lady learned as well as beautiful. She was born at Alexandria. Hypatia was mistress of all the ordinary accomplishments of her sex, familiar with the most abstruse sciences, and made such progress in Philosophy, Geometry, Astronomy, and the Mathematics generally, that she was held as the most learned person of her time. She published commentaries on Apollonius's Conics, on Diophantus's Arithmetic,† and other works. She was chosen whilst very young to succeed her father in the same school, and to discourse from the presidential chair, at a time when Ammonius, Hierocles, and many other very learned men abounded at Alexandria and in various parts of the Roman Empire. Among her pupils, not less eminent than numerous, was the much esteemed Synesius, afterwards Bishop of Ptolemais. But it was not Synesius only and the disciples of the Alexandrian School who admired Hypatia for her virtues and learning. She was held as an oracle by the public, and was consulted by the magistrates on all important occasions. In short, when Nicephorus intended to pass the highest compliment on Eudocia, he thought he could not do it better than by calling her

* (380 A.D.)—380 years after Christ: A.D. being the abbreviation of *Anno Domini*, the year of our Lord.

† 'Diophantus was as great in Arithmetic as Archimedes, Apollonius, or Euclid was in Geometry, and by him was found out that admirable art we call Algebra.'—*Tacquet.*

another *Hypatia.* Whilst Hypatia thus reigned the most brilliant ornament of her sex in the annals of history, she was greatly admired by Orestes, the governor of the city, who, on account of her wisdom, often consulted her. This, together with an aversion which St. Cyril had against Orestes, proved the cause of her ruin. About 500 monks assembled, attacked the governor one day, and would have killed him had he not been rescued by the townsmen—and the respect which Orestes had for Hypatia, causing her to be traduced among the Christian multitude, they dragged her from her chair, tore her in pieces, and burned her limbs.* This shocking and brutal catastrophe was perpetrated in the lent of the year 416, in the time of Theodosius II.†

Pappus, a consummate mathematician, flourished about the end of the 4th century. His work, entitled 'Mathematical Collections,' has transmitted his name to posterity with distinguished lustre.

Proclus, who was at the head of the Platonic School, established at Athens (A.D. 500,) rendered important service to the sciences, and showed great kindness to all who embraced their pursuit.

The age of important discovery was past before the time of Pappus—and science in his time was beginning to decline. The labours of Proclus and a few of his followers were the last expiring efforts

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* It is related by Damasius and Suidas, and clearly proved by such learned men as Bruker, La Croze, and Basnaoe, that St. Cyril incensed the Christian populace against her.—(*Voltaire's* Phil. Dict. art. Hypatia.) St. Cyril was a man of parts, and there is a difficulty in accounting for the inhuman principles that actuated him in this affair.

† For a more particular account of this illustrious victim of fanaticism, see Bossut's Hist. Mat., English Ed., 8vo., 1803.

made in Greece for ancient Geometry. The Alexandrian School still existed, and science might have continued to flourish, had not the conquest of Egypt by the Arabians, who, with the design of spreading their religion and their empire, inundated Christendom in the seventh century, and not only gave it a fatal blow in Alexandria, but also extinguished its now languid existence in Greece.

Throughout all the civilised parts of the world in those times, the Alexandrian Library was known as the treasure-house of literature and knowledge. But neither the members nor the materials of this sacred edifice were respected by the Arabians, who, stimulated perhaps more by religious fanaticism than barbarous ignorance, sacked Alexandria, and massacred the philosophers who were assembled there for the purposes of study from all the neighbouring countries. Their laboratories and instruments were destroyed, and finally the Library itself, the accumulated scientific treasure of ages, containing above 200,000 written volumes, was committed to the flames for the mean purpose of heating the public baths. Caliph Omar, as he gave orders for the annihilation of the books, justified it by the strange, but convenient argument, that—'if the volumes were conformable with the Koran, they were superfluous; and if contrary to it, impious.' With this brutal act of bigotry and devastation may be said to have closed the scientific career of antiquity.

As a partial amends for so odious an outrage against civilization, the Arabians, about a century after, began to cherish the sciences of Geometry and Astronomy, and thus afforded an asylum which preserved them from entire extinction. About the time of their learned prince Almamon, several of their most celebrated works on Geometry were translated into the Arabic. Geber Ben Aphla laid the foundation of the

modern Trigonometry. An elegant treatise on Mensuration was written by Bagdadin; and it appears from his treatise on Optics, that Alharin was a superior geometer. They studied also the more sublime Geometry, improved Trigonometry, and reduced it to a convenient form.

Of the few Persian geometers, Nassir-Edin and Maimon-Reschid were the most distinguished. They both composed commentaries on Euclid. Persian learned men called Geometry the difficult science, and did not make any improvements in it—neither did the Turks nor Hebrews, although they translated some of the elementary works.

In one of the ancient treatises on Astronomy of the Hindoos, called Suryá Sidhánta, there is a correct system of Trigonometry; and hence it is concluded that the Indians had cultivated Geometry before the composition of this treatise. The Chinese were acquainted with the 47th of the First Book of Euclid.

The Romans knew little more of Geometry than the practice of land measuring. Varro, however, composed a treatise on Geometry, and Cicero had some acquaintance and esteem for it too. Vitruvius and Boetius had some knowledge of Geometry, but the latter is better known for his '*Consolations of Philosophy.*'

The learned Bede, who lived at the beginning of the 8th century, was acquainted with all the branches of the Mathematics. Under him Alcuin studied, who was well versed in mathematics, and was preceptor to the celebrated Charlemagne. He was a native of Britain, where, about this time, more attention was given to the study of this science than in any country in Europe.

For nearly two centuries after this period, the study of the science was extinct. No mathematician

of any eminence appeared till the beginning of the 15th century, when, with the general revival of learning, this science was destined to commence a new and more splendid career.

The Italians, by their intercourse with Arabia, became, about the end of the 12th century, the restorers to Christendom of its long-forgotten arts and sciences. Leonardus Pisanus, a rich merchant of Pisa, who had made, in the course of his profession, several voyages to the East, is said to have been the first who disseminated amongst Europeans a knowledge and a love of mathematical studies, after both had disappeared for so many ages.*

About the beginning of the 13th century, Campanus, of Navarre, translated Euclid, and several geometrical works were translated from Greek and Arabic MSS., by Maurolycus, of Messina, who was distinguished for some original and elegant works of his own. Ramus, or La Ramée, an original metaphysical writer, also composed several excellent works on Geometry: and Willibrad Snellius published at the age of seventeen an attempted restoration of an ancient work of much value.

About the beginning of the 16th century the Spanish and Portuguese geometers, John of Royas and Nonnius or Numer, lived. The former was the inventor of the projection of the sphere, and the latter was the first to introduce the Arabic System of Algebra into Europe. About the same time, Wright, a skilful geometrician, invented his chart—improperly called Mercator's chart. At this time, Germany produced the three mathematicians, Werner, Rheticus, and Pitis-

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* A MS., dated 1202, of this celebrated Italian, has lately been discovered by Cossali, a canon of Parma.

cus. Werner made a German translation of Euclid. The Italian geometer, Lucus Valerius, determined the centre of gravity in various mathematical figures, conoids and spheroids.

Kepler was the first to conceive the theory of magnitude being composed of indefinitely small elements of the same kind; as, for instance, that a circle is composed of the surfaces of an indefinite number of very small triangles having the centre for their common vertex, and their small bases on the circumference. By means of this theory, though not very satisfactory in regard to scientific rigour, he extended Geometry considerably beyond the stage in which it was left by Archimedes. Cavallerius improved on the theory of Kepler, and effected a still further extension of Geometry by conceiving lines to be composed of points, surfaces of lines, and solids of surfaces.

Galileo, so eminent for his discoveries in physical science, did little to improve Geometry. He invented the Cycloid, called the Helm of Geometry on account of the discussions that originated concerning its properties. Gregory St. Vincent and Andrew Tacquet were two Flemish mathematicians, the former prosecuted the quadrature of the circle—the latter, the quadrature of surfaces and conic sections, with considerable success. He rendered essential service to the study of the science by subjoining the *Uses* to the propositions in an edition of Euclid's Elements, he published, after the manner of the celebrated French Jesuit, Claude François Milliett Dechales. Blaise Pascal, a Frenchman, (born 1623, died 1662) was equally distinguished for his great literary and scientific attainments, was the author of several valuable researches respecting the properties of the Cycloid; and at the age of sixteen, he composed a complete and elegant treatise of conic sections. But his health was

so much impaired, that soon after he was compelled to suspend his studies for four years; and he ultimately perished at the age of 39, after a lingering illness, in Paris.

Descartes, another eminent Frenchman, (born 1596, died 1650), had a profound knowledge both of mathematical and metaphysical science, was one of the most efficient agents in accomplishing that intellectual revolution which was in progress during his time, and was the inventor of a method which produced a thorough and memorable improvement in Geometry. Before his time the application of Algebra to Geometry, in the ordinary sense of the term, was known, as examples of it are to be found in the works of some ancient geometers. But Descartes invented that more general mode of applying it—that the science of Geometry, instead of being confined within the narrow limits imposed by the complexity and inadequacy of the ancient methods, is now of indefinite extent.

Hüygens, a native of Holland, (1629—1705) was a profound mathematician. He solved a problem of such difficulty, that it had not been previously attempted.

The year 1642, which 'gathered' Galileo 'to his fathers,' gave to the world Isaac Newton, the Archimedes of modern times, or, as he has sometimes been styled, from the number and importance of his discoveries, the Prince of Modern Mathematicians. The fame of Sir Isaac rests chiefly upon the *application* he made of his acquisitions, in explaining the laws or order by which the mechanism of the universe appeared to him to be regulated. His views with regard to this matter have been styled, in honour of him, the Newtonian System of Philosophy, and have been received implicitly by the learned world till nearly the present time.

The impetus given to mathematical pursuits by

the genius of Newton, has descended to our day, to use a phrase of his own, 'with increasing momentum.' For in his train have followed the illustrious names of Barrow, Keel, Gregory, Bishop Horsley (editor of an edition of Euclid's Elements), the Rev. Mr. Lawson, Robison, and many others. Among the most celebrated, rank Leibnitz, Euler,* the Bernouillis, Clairant, D'Alembert,† Thomas Simpson, Robert Simson, the well known restorer, and Playfair, the improver, of Euclid—Young, Leslie, and Madame Agnesi,‡ and Baron Swedenborg.§

It is not necessary to extend this enumeration. The mathematicians and geometers of this generation are known to the public. But it is above estimate how the diffusion of knowledge, one moment dreaded as mischievous, and in the next with the customary inconsistency of folly, pronounced impracticable, will increase the public taste for this science, and create a just and enduring appreciation of this useful order of scholars.

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* Euler lost his sight by intense application, and like Milton, closed his labours in blindness.

† One of the writers in the celebrated French 'Encyclopædia.'

'‡ This lady deserves the most respectful mention. Her Algebraical Works, in quarto, translated by Colson, have been long before the public, and very deservedly esteemed.'—*Harris's* 'Compendium of Algebra.'

§ Swedenborg's 'Principia,' a work declared, only very lately by one of the highest professors in Europe, 'not unworthy of being placed by the side of Newton's.'—*Douglas Jerrold's Mag.*, vol. 2, page 91.

CHAPTER IV.

UTILITY OF MATHEMATICS AS A MEANS OF EXTENDING OUR KNOWLEDGE OF THE PHYSICAL WORLD.

Plato denied the world can be
Governed without Geometry.
Butler's Hudibras.

It is a very trite observation, that human knowledge has been greatly extended by means of this science. Besides the important properties of magnitudes, and wonderful relations of abstract quantities which it has made known, it has unfolded a very extensive range of natural phenomena. It has investigated the principles of theoretical mechanics, the laws of equilibrium and motion of fluids, fixed and elastic. Optics, electricity, and magnetism are its debtors. The theory of acoustics, the propagation of light, and numerous other branches of science, have received great improvements by its agency. Even its reputed conceits have been susceptible of useful application. The ancient doctrine of the Conic Sections, which for 2000 years was an object of mere curious speculation, became, in the hands of Newton, a most efficient means of unfolding the planetary motions.

'Without the aid of rules derived from this science, the navigator, relying on his compass as a guide, could not with safety venture to any considerable distance on his element; intercourse with transmarine regions would be impossible; and, consequently, our knowledge of the globe we inhabit would be very limited. We should probably yet believe that its surface is an extended plane, and that it is supported on pillars; or, as was the opinion of some ancient philosophers, that

its figure is cylindrical, like a drum. Without the aid of this science, our knowledge of celestial bodies would be still more imperfect, and the consequences of our ignorance still more striking. We should still believe that these objects are equally distant from us, and, very probably, that they are distributed on the surface of an extensive crystalline sphere, performing a diurnal rotation about the earth, as the centre of the universe. We should also believe that some celestial phenomena, as eclipses and comets, are certain signs of a conflict of the elements of nature, or that they are the portentous indications of the wrath of heaven, while contemplating to inflict on superstitious mortals some dire calamity, as war, pestilence, or famine.

'How different from these unsatisfactory and incoherent conjectures is that great achievement of this science—the clear and satisfactory exposition, on the most incontrovertible principles, of the complex though sublime and systematic mechanism of the heavens; by which the distances and magnitude of the sun and planets have been measured, and also their weights, and even that of their satellites, ascertained; and by which the masses and distances of some of the stars, or suns of other systems, though inconceivably remote, even in comparison with the great extent of our own system, will probably ere long be determined.'*

An abler pen has celebrated this theme. 'The scientific principles that men employ to obtain the foreknowledge of an eclipse, or of anything else relating to the motion of the heavenly bodies, are contained chiefly in that part of science which is called Trigonometry, or the properties of a triangle, which, when applied to the study of heavenly bodies, is called Astronomy; when applied to direct the course of a

* Bell.

ship on the ocean, it is called Navigation; when applied to the construction of figures drawn by rule and compass, it is called Geometry; when applied to the construction of plans of edifices, it is called Architecture; when applied to the measurement of the surface of any portion of the earth, it is called Land-surveying. In fine, it is the *soul* of science. It is an eternal truth. It contains the mathematical demonstration of which man speaks, and the extent of its uses is unknown.'*

The author of the Geometrical Companion defends the practical uses of Geometry with the ardour of an enthusiast. His examples are happy and forcible.

'A bee, which is an instinctive geometer, finds the use of it in constructing its comb, so as to be the most roomy and durable at the least expense of time and labour. Will any one after this fact doubt the use of Geometry to an intelligent creature?

'Almost all the evolutions of an army consist in converting one parallelogram into another. And the works of a fortified town, or position, are little else than combinations of parallelograms forming bastions, curtains, and parapets.'†

In our 'Uses,' other applications of this science will be seen. Perspective, Dialling, and a variety of the arts, might further be adduced. Literature gathers from it many of its prettiest similes. Dr. Johnson has two examples in his Life of Addison:—'A simile may be compared to *two lines converging to a point*, and it is more excellent as they approach from a greater distance.'

'An exemplification resembles two *parallel lines*, which run on together without approximation, never far separated, and yet never joined.'

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* Paine. † Darley.

Emerson has a fine thought adorned with the vestments of geometry. 'Every natural process is but a version of a moral sentence. The moral law lies at the centre of nature, and radiates to the circumference.'

CHAPTER V.

MATHEMATICS AS A MEANS OF MENTAL DISCIPLINE.

The same excellent persons in the commonwealth of learning who discovered Philosophy discovered also the Mathematics: like twin sisters they came into the world, and like twin sisters there is a natural affinity between them.—*Tacquet.*

It is a mortifying reflection that the soberest of sciences should have been disfigured by the exaggerations of untutored enthusiasm. The Mathematics have been extolled as the only source of logical acumen and cool judgment. Its discipline has been assumed to be the most salutary to which the human powers can be subjected. Some indiscreet votaries have carried its principles out of their legitimate province, and applied them to subjects they were never fitted to investigate. As might be expected, these proceedings have led to the disappointment of those who believed such unwarranted pretensions, and to the ultimate undervaluing of the real and substantial merits of Mathematics. No more than the varying constitutions of mankind admit of one panacea for human ailments, do the variations in human intelligence admit the pretensions of one science to train up the faculties of every individual. Natural Philosophy, Political Economy, Morals, Metaphysics, each has its peculiar discipline by which it addresses peculiar persons, and matures

their powers where Mathematics would entirely fail. The discreet friends of Mathematics never hesitate to allow this, and with the judicious discrimination of Tacquet, care only to claim for it kindred affinity to Philosophy, and the moderate but just honour of its having shed a certain lustre over human manners, and the arts and the sciences at large.

Dr. Arnott, author of the Elements of Physics, has attacked the assumed pretensions of Mathematics, and though he is far from doing it skilfully, it cannot be denied that he had occasion for his reflections. 'There is,' he says, 'no occupation which so much strengthens and quickens the judgment as the study of *Natural Philosophy*. This praise has usually been bestowed on Mathematics; yet a knowledge of abstract Mathematics existed with all the *absurdities of the dark ages*; but a familiarity with Natural Philosophy, which comprehends Mathematics, and gives tangible and pleasing illustrations of its abstract truths, seems incompatible with any gross absurdity. A man whose mental faculties have been sharpened by acquaintance with these exact sciences, in their combination, and who has been engaged, therefore, in contemplating *real relations*, is more likely to discover truth in other questions, and can better defend himself against sophistry of every kind. We cannot have clearer evidence of this, than in the history of the sciences, since the Baconian method of *reasoning by induction* took the place of the visionary *hypotheses* of preceding times.'

Dr. Arnott does not seem an enemy to pretension, provided he is allowed to advance it. The kind of eminence he denies to Mathematics, and which might safely be denied of any science, he does not hesitate to claim for Natural Philosophy. He unhesitatingly asserts that 'there is *no occupation* which so much strengthens

the judgment as the study of Natural Philosophy.' But common experience, in the variety of intellectual aptitudes, teaches us the fallacy of this broad assertion. It is not a disparagement, as Dr. Arnott supposes, that 'Mathematics existed with all the absurdities of the dark ages;' on the contrary, it is no little praise that Mathematics had vitality enough to exist at all under that weight of superstition and barbarism, which buried all other learning in undistinguished ruin.

A geometrician, whose ingenious performances I shall have several occasions to cite, and whose popular pen will give him considerable influence in other quarters, has contributed a eulogium to his favourite science, which cannot be permitted to pass unnoticed.

'Geometry not only sharpens our faculties, corrects precipitancy, and prevents liability to imposition, either from the arguments of others or our own, but it gives us a satisfaction in our knowledge [which we could not otherwise obtain], and thereby enables us to speak with a strength and perspicuity which a vague persuasion of the truth of what we utter could never inspire.'

Mr. Darley affirms too much. He plainly alludes to 'satisfaction' in knowledge in general, and the words I have put in brackets should have been omitted, as men can, for thousands do, 'otherwise' succeed in securing that satisfaction.* It is enough to contend, which

* It is but fair to this writer to state that he has deduced a compliment to Geometry from the biography of distinguished men, expressed with perfect propriety:—

'Alexander, Cæsar, Charles 12th, Buonaparte, and two of the most celebrated military engineers, Vauban and Coehorn, were profound geometers. And it is to their knowledge of this science, the superior excellence of their systems must be attributed.'—*Geometrical Companion.*

would be true, that the study of Geometry is *one* of the modes of educational discipline, and a most approved method indeed. It is contending that it is the only method which discovers vanity and offends good taste.

When a stranger, an enemy, or an indifferent person, chooses to laud Mathematics to the disparagement of other branches of knowledge, the encomium may be tolerated, though by no means imitated. Dugald Stewart, who had no particular predilection for the Mathematics, thus speaks of its study in his Philosophy of the Human Mind :—

'The intellectual habits of the metaphysician afford little or no exercise to that species of attention which enables us to follow long processes of reasoning, and to keep in view all the various steps of an investigation till we arrive at the conclusion. Such processes are much longer in Mathematics than in any other science; and hence the study of it is peculiarly calculated to strengthen the power of steady and concatenated thinking; a power which, in all the pursuits of life, whether speculative or active, is one of the most valuable endowments we can possess.'

The Geologist, the Astronomer, the Chemist, severally have lauded their respective sciences as the most important that can engage the attention of mankind, both in an absolute and educational sense. This is the language of ignorance or vanity, Mathematics need never descend to the disparagement of other sciences. It has its distinctive merits: it can bear any comparison, its province is peculiar, its utility unquestioned, and its method of procedure instructive. These facts are its proper eulogies, and time has failed to obscure, or the envy of ages to diminish, their lustre.

Some of the features of discipline which indisput-

ably characterise mathematical study, have been well depicted by a modern writer.

'In reasoning, as in other arts, we are not masters of what we do, till we do it both well and unconsciously: now this advantage a judicious cultivation of Mathematics supplies. It familiarises the student with the usual forms of inference, till they find a ready passage through the mind, while anything which is fallacious and logically wrong, at once shocks his habits of thought, and is rejected. He is accustomed to a chain of deduction, where each link hangs on the preceding; and thus he learns continuity of attention and coherency of thought. His notice is steadily fixed on those circumstances only in the subject on which the demonstrativeness depends, and thus that mixture of various grounds of conviction which is so common to other men's minds is rigorously excluded from his. He knows that all depends upon his first principles, and flows inevitably from them; and that however far he may have travelled, he can at will go over any portion of his path, and satisfy himself that it is legitimate; and thus he acquires a just persuasion of the importance of principles on the one hand, and, on the other, of the necessary and constant identity of the conclusions legitimately deduced from them.'*

Dr. Olinthus Gregory, in his farewell address to the pupils of the Royal Military Academy, speaks of the study of Geometry as leading to that valued attainment—'*The power of mastering the mind.* If it be desirable to obtain and keep the ascendancy anywhere, it is, surely, *at home*, in the centre of your own intellect and its principles, your heart and its emotions.

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* Thoughts on Mathematics as a part of Liberal Education, by the Rev. William Whewell, Cambridge.

Gain the mastery here, govern within, learn to direct your thoughts to any subject you please, and keep them uninterruptedly to their occupation, till at *your* bidding the labour shall be remitted: then all will be well, for all will lead to a prosperous issue.'

We all know, that if human legs made peregrinations without the consent of the owner, how great would be the hindrance to business, yet how often is thought found roaming the world over when it should be confined at home But the command of the thoughts is as essential to success in thinking as the control of the limbs is to the dispatch of business. Inasmuch as Mathematics imparts this power, it is valuable discipline.

In the same address Dr. Gregory justly remarks, that the power of doing one thing at one time is the hardest and the most important lesson a young man has to learn. He is of opinion that the concentration of attention and good sense included in this habit, is powerfully impressed upon the notice of the mathematical student. The wisdom of this course, as applicable to children as well as adults, has lately been forcibly illustrated by a lady in these words:—'A very quick and clever child made an observation to her governess before me the other day, which had much of truth in it. "How is it, my dear," inquired the lady, "that you do not understand this simple thing?" "I do not know, indeed," she answered, with a perplexed look, "but I sometimes think I have so many things to learn that I have not time to understand."'

If we may trust the testimony of another enthusiast, this science exercises a species of influence of a very high order. 'The moral influence of the Mathematics did not escape the penetration of Plato, who, chiefly on that account, decides that the study

should be cultivated in his "Republic." "Geometry," says he, "is the knowledge of that which is eternal; it disposes the mind to the contemplation of truth, and gives a philosophical habit, which withdraws our thoughts from what is low to what is elevated. Let not the citizens of this renowned Republic neglect the study of Geometry."' Mr. Cooley concludes with observing—'We well know how much it conduces to the easy acquisition of any kind of knowledge, to have previously learned Geometry.'*

But it is far from my intention to rest the reputation of geometrical studies upon declamation, however eloquent, or upon testimony however respectable. An exposition of the Logic of Euclid is the best demonstration of the value of its discipline, and this I shall now attempt.

CHAPTER VI.

THE LOGIC OF EUCLID.

'Among the various guides which have been written for the direction of the understanding, no system of logic has ever been devised better calculated for this purpose than the Mathematics.'—*James Harris:* 'Introduction to Compendium of Algebra.'

Moderating somewhat the absolute tone of Mr. Harris's opinion, it may be taken as expressive of the truth. The ancients considered Euclid their best book of logic. Its main features as such are, that from a few axioms, self-evident, and definitions, agreed upon, new truths are evolved by the perception of the *coherence*

* Cooley's Introduction to the Study of Mathematics.

subsisting between them and the things previously admitted. These truths are added in their turn to the ranks of axioms from which others are to be inferred. Until this day sciences are considered perfect in proportion as they possess these features.* To Geometry belong three principal things:—Axioms, or first principles—Definitions, or explanations of the meanings of the chief terms employed—Postulates, or things required to be granted.

An enquirer must understand himself, and disputants must understand each other, or efforts after conviction will be futile. The geometrician (for we shall use him as the exponent of mathematical procedure) commences by distinguishing what he must assume: for such is the limited nature of human intellect, that all reason must primarily rest on assumption. Accordingly, Euclid lays down axioms which, being, as their name implies, self-evident, become the common point from which disputants can start on the path of investigation. In geometrical treatises, Definitions are usually placed first, but in logical propriety Axioms demand first to be considered, as the assent to definitions is often made by a secret reference to axioms in the thoughts.

AXIOMS.

The first principles, or fundamental axioms commonly used in Geometry are these:—'If a thing be divided into parts, the whole is greater than any one of the parts. Two straight lines cannot enclose a space. All lines drawn from the centre to the circum-

* Mr. James Milne, in a curious work on National Education, has constructed outlines of Political Economy, Metaphysics, and other subjects, on these principles. Though not entirely successful, they are instructive speculations.

ference of a circle, are equal to each other. Two things cannot be both equal and unequal at the same time. Things equal to the same thing are equal to one another.'* The information conveyed by such elementary truisms is necessary: they are the very A, B, C, of the science—the lowest steps of the mathematical ladder, and as essentially useful as the highest.

The patient reference which the geometer makes to these first principles is highly instructive. He never loses sight of them. At his highest altitude he turns to refresh his understanding at those clear fountains of sense—an example which every reasoner may imitate, and an advantage which the investigator in every science may profitably secure to himself.

Dr. South has encouragingly remarked, that the reason of all things lies in a narrow compass.† The geometrician is professionally master of this secret, so true of every rational theme of human speculation. An Iliad of meaning lies concealed in the nut-shell of a fundamental axiom. Of what importance is this truth to the enquiring public of our day?

'How many readers are there who would not be glad of attaining to knowledge the shortest way, seeing the orb thereof is swollen to such a magnitude,

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* It strikingly evidences the intimate connection between Logic and Geometry that this axiom and its converse are found at the root of Aristotle's Syllogisms. Syllogisms were deducible from these two principles—First: things which agree with the same thing, agree with one another. Second: things whereof one does, and one does not agree with the same thing, do not agree with one another.—*Bruce.*

† The reason of things lies in a narrow compass. Most of the writings in the world are but illustration and rhetoric, of little consequence to a mind in pursuit after the philosophical truth.—*Dr. South.*

and life but such a span to grasp it? How many who have not some curiosity to know the *foundations* of those tenets upon which they so securely trust their understanding? or where the *footsteps* of those opinions and precedents may be found, which have given direction to so many modern performances? *

No science is fully mastered till the principles on which it is based are clearly seen. When these are lost sight of, all subsequent reasoning is confused. The geometrical student is early and much accustomed to keep these important considerations before him. The most abstruse proposition is an axiom to the mind that perceives its truth, and the most complicated science resolves itself into a very simple affair when its first principles become familiar to the understanding. These considerations reduce learning to an art, and give tone, power, and comprehensiveness to the understanding, which fit it peculiarly for the acquisition of knowledge.

Ordinary experience in learning and teaching must frequently impress the truth of these remarks on careful thinkers. When a pupil has a voluminous grammar put into his hands, he feels, with Dr. Johnson, that a great book is a great evil. But when the first principles of language are clearly seen, the difficulties vanish and the study is mastered.

This is not only the secret of the acquisition of knowledge, but also the agent for retaining it. Around these first principles, as around a standard, the thoughts naturally associate. Touch but a remote chord of any question, and it will vibrate to the central principles to which it has once been well attached—every relative impression owns a kindred connection, and the moment one is attacked, it, like a faithful sentinel, arouses a whole

* Oldys.

troop, who, marshalled and disciplined, bear down with their whole strength, and challenge the enemy.

DEFINITIONS.

That confusion in the varying meaning of terms, which reigns through most subjects that agitate mankind, is unknown in Geometry. The Definitions at the very threshold for ever fix the question of sense. Every leading term is defined. There are no words used in Geometry whose meanings are so much alike, that the ideas for which they stand may be confounded together. If there is any distinction between the meanings of two terms, it is a total and a complete distinction. Locke, in illustration of this point, remarks—'The idea of *two*, is as distinct from the idea of three, as the magnitude of the whole earth is from that of a mite; this is not so in other modes, in which it is not so easy, nor perhaps possible for us to distinguish between two approaching ideas, which yet are really different; for who will undertake to find a difference between the white of this paper, and that of the next degree to it?'

The meanings once fixed in Geometry are ever after adhered to: the same word is for ever used in the same sense. For instance, the term *right* signifies *straight*, and *straight* is its everlasting meaning in Geometry. In political philosophy, *right* is a well-known stumbling block, which perpetually overthrew the fine genius of Burke. 'Natural right,' 'inherent right,' 'moral right,' and 'political right,' are only a few of the rights which every now and then some puzzled politician will pronounce wrong. It perhaps is not possible to restrict popular phrases as we can scientific terms. But the geómetrician, perceiving the advantages of doing so, acquires the taste of precision, and takes care (which is practicable with

everybody) to define the *principal* terms he may employ in any given dissertation.

POSTULATES.

With that precision from which the geometer never departs, he defines his mode of procedure. For this purpose he puts up his Postulates or petitions to be permitted a certain latitude of action. This latitude is never exceeded. Thus neither his language nor his practice is left open to cavil. Admit my axioms, agree to my definitions, and grant my Postulates, says the independent geometer, and I will engage to demonstrate every proposition I lay down.

A proposition to be proved, or a problem to be solved, have common parts—the Proposition, Enunciation, Construction, Demonstration, Conclusion. (These parts, for the assistance of the uninitiated, are clearly distinguished in the following pages.) *

The proposition being plainly laid down, the enunciation fixes its meaning, the construction provides for its proof, and a rigid demonstration leads the way to the conclusion. Every step has its distinct reference, establishing its legality and fairness. There is no attempt at evasion—no manœuvre, no equivocation. This daring self-reliance cannot fail to impart manly qualities.

The student is distracted with no impatient attempts to accomplish two things at once. One thing at one time is the order of Geometry. In the language of the propositions, all circumlocution is avoided. What is intended is straightway expressed. No digressions

* One who does not understand the principles of Euclid's Demonstrations, knows absolutely *nothing* of Geometry: unless he attain this point, all his labour is utterly lost.—*Dr. Whately.* Preface to Logic,' p. xxi.

are endured. Nothing is admitted but that which has been sanctioned, or established, or is relevant. Nothing is asserted but that which is intended to be demonstrated. It is impossible that these methods of procedure can be perceived and practised by the student without his acquiring valuable lessons, applicable to other subjects, and to much of the conduct of life.

The reader, in this place, must be cautioned against the language in which the Logic of Euclid is sometimes eulogised. Here is an extract from a recent journal:—'Never does the mathematician say, such a proposition is true because it is probable; or, because the balance of the *pro* and *con* arguments preponderates in its favour; or, because history records it, and tradition affirms it; or, because the holy St. This declared it, and the learned Philosopher That believed it; or, because infallible *I* assert it. No! HE takes a nobler stand, HE claims more lofty ground; his words are—"I have actually and incontrovertibly PROVED so and so to be a fact; disbelieve me, *if you can;* refute me, *if you are able.*"' This is the folly that encourages the contempt with which this fine study is sometimes treated.*

The reason why the geometer neither refers to probability nor authority is, that the nature of his science enables him to do without them. In the affairs of mankind there are numerous vital questions, such as those of law and legislation, which, from the nature of things, must be decided by probability: by *pro* and *con* arguments, and such decisions are as respectable and as salutary, if not quite as safe, as the conclusions of Mathematics. Those speculations

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† It was this thoughtless arrogance which provoked the partial dictum ~~severe exclamation~~ from a recent critic—'This logical and mathematical school of literature can only hold its sway with narrow minds and in an unimpassioned age.'—*Douglas Jerrold's* 'Shilling Magazine,' vol. 1, p. 572.

—and others, curious and important ones, in which authority and personal conviction for a long time form the only accessible evidence—are not to be rejected on that account, but to such evidence is to be allowed its proper weight. Let not flippant arrogance, rejoicing in the strength of Geometry, depreciate the arguments peculiar and necessary to other subjects. The modest mathematician learns the lessons of valuing strict demonstration wherever it can be had, and rightly refuses conjecture where certainty is to be obtained. He knows, if he has general information, that every proposition cannot be proved after the manner of Geometry, but he has a right to expect that what is attempted to be demonstrated in the same way shall be demonstrated with equal strictness.

Geometry is theoretical and practical: demonstration, direct and indirect: and reasoning is synthetical and analytical. Grammar is an illustration of the synthetical; Chemistry of the analytical. In Grammar we begin with letters—these are combined into syllables — syllables into words — words into sentences—and sentences into discourses. This is synthesis, or putting together. In Geometry, the ~~grand~~ element of the whole superstructure is a point, a point becomes a line, a line a surface: and of these solids are composed. Geometry is synthetical. In Chemistry, water is decomposed and resolved into its elementary principles by means of analysis, or taking to pieces. This is a similar demonstration, though obtained by a method the reverse of the former. There is a third method—namely, Induction, which Hume, with more precision than Locke, calls the doctrine of proofs. Induction is inferring from several particular facts, or propositions, a general truth. All these modes have important uses in different investigations. Geometrical reasoning is inductive. The

Axioms and properties of Definitions are the facts from which first conclusions are deduced. Every proposition established, is an addition to these facts, which become the increased bases of inferences. From coherency between ideas, the geometrical logician proceeds to coherency between facts, and from the entire body of facts before him he discovers the coherency of his general inference. Reason is the ability to perceive coherences. Reasoning on the abstrusest theorems is nothing more than after having arrived, step by step, at a remote truth, discovering its connection with preceding facts in the same chain. Geometrical problems alarm the uninitiated, who are not aware that a little attention conquers the simple ones, and a little patience the most difficult. One in the secret thinks them all equally easy, because he sees they are all equally connected. So far as geometrical study affords practice in mastering these consecutive coherences, it may be considered as aiding the development of reasoning power.

The 'First Book of Euclid' is a perfect illustration of geometrical logic. It embraces the foundation of all the subsequent books. They deal with new propositions and advanced reasoning, but the principle of procedure is the same as in the 'First Book.' So when the inquirer understands that, he comprehends the principle of them all. This 'First Book' furnishes general principles of reasoning, which will carry the student, without other assistance, through every subject to which his attention can be called. He has only to use them as initiative guides—giving him a taste for strictness in every application—not as fettering him to one species of evidence and inference. The discretion here alluded to is that so well enforced by the author of *Hermes.* 'When Mathematics, instead of being applied to *exemplify* Logic, comes to supply its place,

no wonder if it pass into contempt. For when men, knowing nothing of that reasoning which is universal, come to attach themselves for years to *a single species*, a species involved in lines and numbers only, they grow insensibly to believe these last as inseparable from all reasoning, as the poor Indians thought every horseman to be inseparable from his horse.'

CHAPTER VII.

NATURAL GEOMETRY.

MR. MACAULAY, in his historical Essay on Lord Bacon, has some ingenious remarks on the natural origin of the species of Induction with which Bacon's name is associated. Similar opinions have been expressed respecting the Syllogism of Aristotle. One of the ablest opponents of John Locke has some profitable speculations of the same kind concerning reasoning in general, all bringing Science, Art, and Logic home to nature, and resting their foundation there. This is the most encouraging aspect of learning that can be presented to the notice of a pupil. No treatise of an educational nature should be without such a feature. If an inquirer once perceives that the germs of a desired art are familiar to him, he makes an important step in advance—he gains confidence. He finds that he has only to systematise and extend his actual knowledge. The barrier of exclusiveness is broken down, and he enters the common ground of the initiated. A geometrician before referred to, has furnished some interesting illustrations of this truth with respect to his favourite science, accompanied by not less useful remarks.

'Suppose Codrus died, and left his field, nearly a square, to be divided among his four sons, Caius, Gaius, Titius, and Tatius. A little consideration would show that it might be done by drawing lines from the middle of one side to the middle of the other, as A B, C D, (see fig. 1). Thus there would be four squares of one size for the brothers. The middle point would perhaps be found by *stepping* along one side, and *half* the number of steps back. Or with a line or a rod. Such modes would be practical, and such as would be suggested to the simplest mind—for the rudiments of Geometry must be in the human mind before the art itself could be formally known as a system.

'Suppose the field to be *three cornered* (see fig. 2). A little more (though not much) sagacity would perceive that its division would be effected by drawing a line from any corner D of the field to the middle point A of the opposite side—and from A to C and B, the middle points of the remaining sides.

'But if the field had been of the unmanageable form as fig. 3, something more than common judgment would be necessary to subdivide *this* into four equal portions. Something would be wanted beyond what the senses or uncultivated reason could furnish, to recognise the equality of the parts A, B, C, D, when the figure *had* been accidentally divided as required. Then men would begin to consider how they might determine the equality or non-equality of differently shaped figures. They would, as we are told was the practice of the early philosophers, draw figures in sand or dust to investigate their proportions: first, because it was useful, and afterwards, because curious. This would be land measurement in miniature. The next removal would be from the sand to the slate, from the dust to the board, and from these to parchment or paper. What before engaged peasants without doors, would now employ philosophers within: what was rude, vague, and unsatisfactory, would now become accurate, definite, and conclusive: what was done experimentally with the rod and the tape, would now be determined rationally with the rule and compass. Behold the gradual transformation of the practical into the abstract art—of primitive into refined Geometry!

'Ask a *ploughman* how he manages to keep his furrows

parallel, and he will tell you, by driving his team and directing the share, so that each new furrow shall keep *at the same distance* from the last, throughout the whole length. In like manner the school boy rules his copy *straight* by keeping each end of the ruler at the same distance from the last line ruled. Thus is the *doctrine of parallels* found inherent in the commonest minds.

'Let us by all means get rid of the scholastic notion, that Geometry is a thing totally distinct from, and independent of, the material world—irrelevant to the ordinary business of life, and uncongenial with the common trains of human reflection: that it is, in fact, an ethereal system, not refined from the grossness of terrestrial conceptions, but absolutely free from all intermixture or affinity with what we see or know about earthly matters: and that it is drawn from the skies, as Socrates drew his moral philosophy, and is a study for those alone who agree to call each other "purely intellectual beings." No such thing. It is only the simplest and most certain of our ideas, gathered from practice and experience, rendered accurate by strict circumspection, and combined with elegant ingenuity into a system differing from others of human invention only in being more perfect. So far is it from being anything of a supernatural or occult science, that a man of strong judgment might, in a popular way, form to himself a system of Geometry almost without knowing how to *read*. In fact, a great many illiterate men *do so*. What a powerful system of natural Geometry must the celebrated Watt have organised in his mind! He was ignorant of this science (that is, of the written word of it), but he was as well acquainted with the common truths of it as the most deeply initiated mathematician, or he would never have made his great discoveries, which involve those properties.' *

A little reflection on the Elements of Geometry, as displayed in the following pages, will further show that the principles of the science have their rise in human experience. The 'cutting off the corner' of a street in walking evinces an intuitive knowledge of a

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* Abridged from Darley's *Geometrical Companion*.

geometrical theorem. The elements are common to all comprehensions—Science develops and perfects them.

CHAPTER VIII.

PRACTICE AND THEORY—OR, THE DISTINCTION BETWEEN PRACTICAL GEOMETRY AND PURE MATHEMATICS.

Pure Geometry is theoretical. Euclid's Elements are considered pure Mathematics. In this species of Geometry everything is done in accordance with the Postulates of Euclid. No instruments, nor compass, nor rule, are used, and the results arrived at are all abstract and unapplied.

Practical Geometry, on the other hand, is seen in the constructions connected with the Propositions, and in the 'Uses' annexed to our 'First Book.' In Practical Geometry, compasses, sections, rules—all species of instruments are employed. It is the application of Geometry to the arts of life.

In pure Geometry something of the practical certainly appears in the 'Constructions' of certain of the Problems and Propositions, but is employed no farther than is necessary for the development of the demonstration—the practical is subordinate, while in Practical Geometry it is the essence. Pure Geometry chiefly demonstrates that a given Proposition is true, or that a given thing can be done. Practical Geometry, on the other hand, teaches all convenient methods of practice, and *applies* it to any or everything to which it can be useful. Such is the aptitude of human intelligence for reality, that when we assume to apply only the Postulates of Euclid, we think of the instruments; but this difference still remains—in Practical Geometry the rule and compass are in the *hands,* in pure Geometry only in the thoughts.

In this work it is attempted to exhibit the 'Beauties' of Euclid's Elements *theoretically*, and the 'Uses' *practically*—because where the theory only is exhibited the question is heard—'It is all very beautiful, but what is the USE of it?' If only the practice is shown, the remark arises—'I see it is so, but I do not see WHY.'* The question and the remark are both proper, and both should be met. These distinctions rule throughout Mathematics. Whatever chiefly pertains to demonstration, and shows that a certain thing *can* be done, or that a certain result *will* ensue, belongs to the province of pure Mathematics—while in Practical Mathematics attention is mainly devoted to modes of operation and to the application of principles. In the pure department, all is abstraction and theory—in the other, all application and practice.

CHAPTER IX.

EXORDIAL ADDRESS TO THE STUDENT.

MORE than 2,000 years have rolled away since the entire Elements were given to the world. A halo of interest, peculiar to no other work known among men, surrounds it. It arose in the morning of Science, flourished among the glorious Geometers of old, kept alive the sacred fire of Learning, when almost the whole earth was without one genial ray, and it now shines with undiminished lustre amid the splendour of modern discovery. It has resisted the attacks of countless critics, great and clever men—has supported the weight of innumerable commentators; still it remains the most perfect work of man, and the finest monument of his reasoning powers. It has been translated into all languages, has been taught for centuries in every mathe-

* Ritchie's *Principles of Geometry.*

matical school of eminence, and is now reputed the best introduction to that science extant.

It may safely be asserted, that as Grammar is the key of literature, so is Geometry the key of the sciences. Without exaggeration, it has been contended that 'the only road to accurate and independent knowledge of most of the sciences lies through the study of Mathematics.'* Euclid is the great schoolmaster of the sciences. His Elements are the pass-word into their dominions. 'Without a sufficient knowledge of Mathematics, that great instrument of [all] exact inquiry, no man can ever make such advances in the higher departments of science as can entitle him to form an independent opinion on any subject within their range.'†

Amasis wondered at the skill of Thales when he measured the Pyramids by means of their shadow. Neither the power nor station of Amasis could place him on a level with the distinguished geometer. Ptolemy, another Egyptian king, is said to have asked Euclid if there was no easier mode of becoming a geometrician than by studying his Elements. 'There is no *royal road*,' was his memorable answer. The meaning of which is, that the acquisition of no single idea can be made in a *royal* way. Only one path was open to Ptolemy and to mankind. Science owns no idle votaries. The only condition on which it yields its secrets is that of attention.

Do not fall into the error of vanity, and neglect learning under the impression that it courts you. It does indeed invite you, but it is in the language of independence. It is a benefactor, and never descends to supplicate the acceptance of its favours. It bestows honour and power on those who seek its wealth, but it never yet conferred a gift on any but the earnest.

Revolve the last words of the venerable professor of Mathematics (Olinthus Gregory)—'In your mental and scientific pursuits; *define* to yourselves clearly the *purposes* which you have in view; see to it that they are in no way incompatible with the nature and the *duty* of man, and you cannot direct your aim at too high a point.'

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* *Review for the Many.* † Sir John Herschel.

THE FIRST BOOK OF EUCLID.

SECTION I.

DEFINITIONS.

1. A *Point* is that which has no parts, or which has no magnitude.*

2. A *Line* is length without breadth.†

CORALLARY.—The extremities of a line are *Points:* and the *intersection* of one line with another is also a *Point.* [Thus (fig. 4) the place A is a point].

3. *Right* or *Straight* lines are such that cannot coincide in any two points without coinciding altogether.‡ [This is substituted for Euclid's, that the Corollary may appear more evident].

COR.—Hence two straight lines cannot enclose a space. Neither can two straight lines have a common segment—that is, they cannot coincide in part without coinciding altogether. [The 14th prop. will illustrate this].

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* A *Point* is that which has no parts.—*Dechales.*

A *Point* is that which has position, but not magnitude.—*Playfair.*

A *Point* is the extremity of a line having no dimensions of any kind—neither length, nor breadth, nor thickness.—*Geom. Lib. Useful Knowledge.*

A *Point* is a mark in magnitude, which is (supposed to be) indivisible.—*Zeno.*

A *Point* is the beginning, as it were, of all magnitude, as unity is of number.—*Tacquet.*

A *Point* occupies no space.—*Legendre.*

† A *Line* has length only, and wants all breadth, inasmuch as it is understood to be produced by the flowing of a point.—*Tacquet.*

‡ A *Right* line is that whose extremes hide all the rest.—*Plato.*

A *Right* line is the shortest of all those that can be drawn between two points.—*Archimedes.*

4. *Parallel* straight lines are such as are in the same plane, and which being produced ever so far both ways do not meet.* (See fig. 5).

5. A *Superficies*, or surface, is that which has only length and breadth.

Cor.—The extremities of a superficies are lines, and the intersections of one superficies with another are also lines.†

6. A *Plane*, or plane surface, is that in which any two points being taken, the straight line between them lies wholly in that superficies.‡

7. A *Plane Angle* is the inclination of two lines to one another, which meet together, but are not in the same right line.

[The two lines, which, meeting together, make an angle, are called the *legs* of that angle. A B, A C, (fig. 7) are the *legs* of the angle B A C. The point A, at which the legs meet, is called the vertex of the angle. An angle may be designated by a single letter, when its legs are the only lines which meet at its vertex. But when more than two lines meet at the same point, in order to avoid confusion *three* letters are employed to designate the angle at that point—the letter which marks the vertex of the angle being always placed in the *middle*. Thus (fig. 6) the lines G C and E C meeting together at C, make the angle G C E, or E C G: the lines G C, E C, are the *legs* of the angle, the

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* Lines are *Parallel* to each other if the perpendiculars be tween them are all equal to each other.—*Archimedes.*

Newton, in Lemma 22, Bk. I. of his 'Principia,' says that parallels are such lines as tend to a point infinitely distant.

† A *Superficies* has two dimensions, and is understood to be produced by the flowing of a line.—*Tacquet.*

‡ A *Plane* surface is a surface which is *perpetually even*, or *flat* throughout its whole extent.—*Darley.*

(On such *planes* all the figures in the First Book are supposed to be constructed).

A *Solid* is a magnitude having the three dimensions of space: length, breadth, and depth. Any one of the six sides of a *Cube* is a *surface*, or *superficies*—an edge is a *line*—a corner a *point.* (See fig. 30).

point C is the *vertex*. In like manner may be named the angle E C A, which is the sum of the angles G C E, G C A: and so of the other angles round the same point.

When the legs of an angle are produced (or continued) beyond the vertex, the angles made by them on both sides of the vertex are said to be *vertically* opposite to each other: thus, since G C is continued to D, and E C to F, the angles G C E, D C F, are said to be *vertically* opposite to each other. And so of the other angles on each side C.

It must be understood, that by an angle is not meant the *surface* between the lines which form it. For though the surface be increased by producing the legs, (to D and E, for instance, in fig. 7), the angle will still remain the same in magnitude. By an *angle*, in fact, is meant the *degree* of *width*, or *separation* between the lines that form it: thus, the *opening* between B A C is greater than the angle F A C.]

8. When a straight line, standing on another straight line, makes the adjacent angles equal to each other, each of these angles is called a *Right Angle*, and each of these lines is said to be at *right angles* to, or *perpendicular* to the other.* (See fig. 8).

9. An *Obtuse* angle is that which is greater than a right angle. (See angle C B A, fig. 49).

10. An *Acute* angle is that which is less than a right angle. (See angle A B D, fig. 49).

11. A *Figure* is that which is bounded by one or more lines. (See fig. 9). The space enclosed within a figure is called its *area*.

12. A *Circle* is a plane figure bounded by one line called the *circumference*, or periphery; of which all lines drawn from a certain point within the figure to the circumference are equal to one another.† (See fig. 10).

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* A carpenter's square is a 'right angle.' Pythagoras is said to have been the inventor of the Square.

A perpendicular line inclines neither to the right hand nor to the left. From this circumspect idea probably arose the precept of the poet—

'Far from extremes a *middle* course is best.'

† Till the time of Plato this was the only curve admitted into Geometry.

13. That point is called the *Centre* of the circle.

14. A *Diameter* of a circle is a straight line drawn through the centre, and terminating on both sides in the circumference.

15. A line drawn from the centre to the circumference of a circle is called the *Radius*, or *Semi-diameter*.

[*Radii* is the plural of radius].

16. *Rectilineal* figures are those contained by straight lines.

17. *Trilateral* figures, or *Triangles*, by three straight lines. (See fig. 11).

18. *Quadrilateral* figures by four straight lines. (See fig. 12).

19. Of three-sided figures: an *Equilateral* triangle is that which has three equal sides. (See triangle A C B, fig. 31).

20. An *Isosceles* triangle is that which has two equal sides. (See triangle A B C, fig. 37).

21. A *Right-angled* triangle is that which has a right angle.* See triangle A E D, fig. 136.)

[The angle opposite the base is called the vertical angle. A side of a triangle, considered in reference to the angle opposite, is said to *subtend* it, or be the *subtense* of the angle. When the side of any triangle is produced, as B C to D, (fig. 14) the angle A C D, made by the produced part C D, with the other leg A C, is called the *external* angle. And the angle A C B, adjacent to the external angle, is called the *internal adjacent* angle. The other two angles of the triangle are together called the *internal remote* angles: and of these, that of which the produced side is a leg, namely, A B C, is the *internal opposite*, and the other, namely, B A C, is the *internal alternate* angle.]

22. Of quadrilateral figures: a *parallelogram* is that of which the opposite sides are parallel. (See fig. 15).

23. The straight line which joins the opposite angles of a quadrilateral figure is called a *Diagonal*.

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* Any side of a rectilineal figure may be called the *Base*. In a right-angled triangle (see fig. 13) the side *a c*, opposite the right angle is called the *Hypotheneuse*, either of the other two sides *a b*, *a c*, the Base, and the one not taken the perpendicular.

24. The *Altitude* of a triangle or a parallelogram is a perpendicular drawn from the opposite angle or side, upon the base, as *a b*, *c d*. (See figs. 16 and 17.)

25. Of parallelograms: a *Square* is that which has all its sides and angles equal. (See fig. 97).

26. An *Oblong*, or *Rectangle* is a four-sided figure, that has equal angles but not equal sides. (See fig. 23). [When a four-sided figure has equal angles, they are always *right* angles. Every 'square' is a 'rectangle,' but every 'rectangle' is not a square.']

SECTION II.

POSTULATES.

(Things required: from the Latin *postulo*, to require).

1. Let it be granted that a straight line may be drawn from any one point to any other point.
2. That a terminated straight line may be produced to any length in a straight line.
3. And that a circle may be described from any centre with any radius or interval.

SECTION III.

AXIOMS.

(Authorities, or things having anthority: from a Greek word. An *Axiom* is a self-evident proposition).

1. Things which are equal to the same things, are equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be taken from equals, the remainders are equal.
4. If equals be added to unequals, the sums are unequal.
5. If equals be taken from unequals, the remainders are unequal.

6. Things which are doubles of the same thing, are equal to one another.

7. Things which are halves of the same thing, are equal to one another.

8. Magnitudes which coincide with one another, (that is to say, which fit together so exactly, that every point of the one lies on some point of the other,) are equal.

9. The whole is greater than its part.

10. All right angles are equal to one another. (Legendre demonstrates this).

11. Two straight lines which intersect each other cannot be both parallel to the same straight line.

[All these Axioms are not required in the First Book, but not being numerous they are inserted as samples of the *assumptions* of Geometry].

SECTION IV.

DEFINITIONS OF TERMS.

A *Theorem* is a truth which becomes evident by a train of reasoning called a *Demonstration.*

A *Problem* is a question proposed which requires a solution.

A *Lemma* is a subordinate or minor truth, which is established in order to be employed in the demonstration of a theorem, or in the solution of a problem.

A *Proposition* is a portion of science. It is a common name applied indifferently to theorems, problems, and lemmas. A proposition is a simple statement respecting any subject.—*Jas. Milne.*

A *Hypothesis* is a fact assumed without proof. (Thus, when it is affirmed that in an Isosceles triangle the angles at the base are equal, the *hypothesis* of the proposition is that the triangle is Isosceles, or that its legs are equal).

A *Construction* is the addition made to, or change made in, the original figure, by dividing or drawing lines, in order to adapt it to the argument of the demonstration, or the solution of the problem. The condition under which